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Addressing the Opioid Crisis in the United States and Modeling its Proposed Interventions

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Abstract

A novel application of compartmental modeling is used to quantitatively study the impact of call centers on influencing an individual's mindset as they begin or attempt to recover from opioid addiction. The opioid epidemic in the United States is a complex public health emergency that has affected millions of Americans; according to the CDC's U.S. Drug Overdose Death Rate Maps on age-adjusted drug overdose mortality by state, West Virginia has been particularly affected by the epidemic compared to other states. This project studies the effectiveness of call centers in increasing active response and recovery rates in relation to opioid use. An active response is characterized by an open mindset and acceptance of help from others, while a passive response is defined by a closed mindset and an unwillingness to believe in the real possibility of recovery. Since these mindsets are qualitative in nature, we offer a quantitative definition using data from HELP4WV, a call center located in the state of West Virginia. We then construct an SIR-type model that mimics the drug-using career on the basic assumption that the drug using population can be compartmentalized into five distinct groups: susceptible, using, assertive, passive, and recovered. The model presented is used to study the effect of call centers on increasing the use of an active mindset in recovery in order to increase long-term recovery rates. Our results derive the basic reproduction number of the system, which is interpreted to mean that so long as the sum of the rates of cessation of addiction are less than the rate of developing an opioid use disorder, the epidemic may be controlled. Existence of at least one endemic equilibrium is proven under specific initial state conditions.

1 Introduction

In this section, we contextualize the topics addressed in the rest of the paper. Section 1.1 includes a brief overview on the recent history and current state of the opioid crisis in the United States. Then we focus on the state of West Virginia in comparison to the national state of affairs. Section 1.2 provides a brief discussion of two particular models which informed ours, laying a basic foundation for understanding the model we developed.

1.1 A brief discussion on the opioid epidemic in the United States

While opioid abuse has a long history in the United States, extending at least to the Civil War era [21], there is a current epidemic of abuse and overdose deaths that has affected millions of Americans over the past 30 years. The first wave of this crisis began in the 1990's when healthcare providers began prescribing analgesics such as methadone, oxycodone and hydrocodone to their patients at increased rates under the false assurance that the drugs were not highly addictive [36]. While the epidemic began to unfold, the global price of heroin was steadily decreasing; this led to a shift from difficult to attain prescriptions towards relatively easier to attain, illicit drugs being the prominently abused form of opioid in the United States [30]. Recently, the country has seen a new wave of addiction and overdose deaths unlike anytime previously, wherein synthetic drugs such as fentanyl are being used, knowingly or not, in conjunction with psychostimulants such as cocaine and methamphetamine [25].

There are several reasons that the opioid crisis remains ongoing in the United States. For one, there is significant economic motivation [23], [35] for those who sell drugs. Additionally, per the nature of addiction, an individual who uses drugs habitually will eventually build a tolerance to the effects and seek either more quantity, stronger drugs, or both. Those who use controlled substances [11] such as prescription opioids may consequently turn to illicit drugs such as heroin in order to achieve a better high. As drugs with even higher potency, like fentanyl, have become more available in the United States, dealers have begun to mix the stronger substances with heroin to increase the intensity of their product. Specifically, fentanyl is up to fifty times stronger than heroin; the 2mg lethal dose is hardly visible to the naked eye [9]. Illicit drugs are also often adulterated when bought on the street [12]. That is, an individual intending to consume one drug may not have any way of knowing whether the product they bought is laced with other drugs such as fentanyl, cocaine, or methamphetamine. The most recent wave of the opioid crisis, impacted by the factors above, has resulted in millions of Americans living with a substance use disorder (SUD), many of whom are specifically diagnosed with opioid use disorder (OUD) [32], every year. An individual with a SUD (such as an OUD) meets at least two criteria defined by the The Diagnostic and Statistical Manual of Mental Disorders, Fifth Edition (DSM-5) [1].

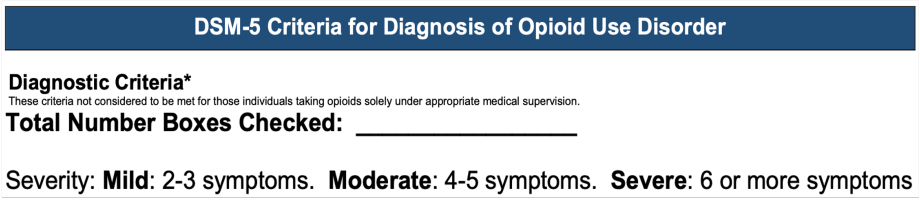

Figure 1: Diagnostic and Statistical Manual of Mental Disorders, Fifth Edition

In parallel with national efforts, West Virginia continues to struggle to suppress the opioid epidemic within its borders. In fact, the state is often considered the epicenter of the crisis in the United States and consistently ranks among the highest in rates of opioid abuse and deaths in the nation, whether it is natural, semi-synthetic, or synthetic drugs being discussed [40], [10]. West Virginia has one of the highest rates of opioid prescription, a leading cause of new addiction [43], [32], which could be associated with the state's high levels of heavy manual labor jobs. These occupations often inherently possess a high risk of serious injury. The state also has among the highest rates of unemployment - another factor that is often linked to drug abuse [10]. Located entirely in Appalachia, West Virginia public transportation is lacking in quality, connectivity and accessibility [40], leaving much of the state population geographically isolated from the healthcare resources they need. If an individual does not have their own transportation, accessing healthcare may not be possible; even if an individual owns a means of transportation, their family, work or health situation may limit the feasibility of access. This demographic and geographic position of the state - predominantly poor, White, blue collar laborers living in a mountainous terrain - is a key factor of it's epicenter status.

1.2 SIR-type models

The most common basic epidemiological model used today is the Kermack-McKendrick (1927) SIR model for the spread of disease. Developed by physician Anderson Grey McKendrick and biochemist William Ogilvy Kermack, the model separates a closed population into three distinct groups: susceptible (S) individuals who are unaffected by the infection, infected (I) individuals, and recovered (R) individuals who have recovered or died from infection. The basic assumptions made by the authors are that the population is constant in size besides disease-related deaths; all individuals are initially equally susceptible to the disease; and complete immunity is gained after becoming ill once [22]. The goal for Kermack and McKendrick (1927) was to gain better knowledge of the factors which control the spread of contagious infections to eventually determine whether a particular epidemic ceases due to the complete depletion of the susceptible population or if there is a case wherein many susceptible individuals remain unaffected by the disease when the epidemic comes to an end. To reach such conclusions, the authors develop a system of ordinary differential equations that effectively describe the movement of individuals to and from each defined compartment. Though Kermack and McKendrick (1927) did not explicitly discuss it, one of the

most important conclusions from SIR models today is the basic reproduction number, R_0. This threshold is defined as the expected number of new infections during the infectious period caused by a single infected individual in an otherwise wholly susceptible population. When $R_0 < 1$, an epidemic is expected to die out. When $R_0 > 1$, the epidemic will not be contained [6].

Drug abuse has been studied extensively using SIR-type models during the last 50 years [4], [14], [27], [28], [29], [31], [37]. This is because mutually exclusive compartments may be defined (a basic example could be (S) susceptible, (A) addicted to drugs, and (R) recovered) and because the spread of drug use is most often caused by contact between drug users and non-drug users [20] [44]. Two particular works stand out in the field and inform the research presented in this paper: White and Comiskey's (2007) heroin epidemic model [41]and Battista et al.'s (2019) model on prescription opioids [2].

White and Comiskey (2007) seek to address the persistence in Ireland of heroin users, characterized by habitual use which disrupts the well-being of the individual, their family, or the society. The authors extend existing research on dynamic disease modeling to the drug using career (initiation, habitual use, treatment, relapse, and recovery) in order to provide a novel application of ordinary differential equations to the spread of habitual drug use in their model. White and Comiskey (2007) define 3 compartments, similar to the Kermack and McKendrick (1927) model described above. Susceptible (S) individuals have not used drugs habitually before, drug users not in treatment (U_1) include initial and relapsed drug users, and drug users in treatment (U_2) are analogous to a recovered group with the exception that they may relapse to untreated drug use. The research lies on the basic assumption that drug use spreads through a population similarly to an infectious disease. Other prominent assumptions include that the population is of constant size, there is homogeneous mixing of individuals, and that all members of the population are equally susceptible to drug addiction. Finally, the authors assume that individuals in treatment are using drugs, drug users not in treatment are infectious to the rest of the population, and drug users in treatment are not infectious to the susceptible population [41].

The basic reproduction number R_0 is derived to be

$$R_0 = \frac{\beta_1}{\rho+\mu+\delta_1},$$

which tells us that when the probability β_1 of becoming a drug user exceeds the sum of the probabilities of leaving the drug using compartment, we can expect an epidemic. White and Comiskey (2007) then use a Jacobian matrix to analyze the disease free equilibrium (DFE) and find that, since the eigenvalues of the Jacobian are real and negative, it is locally asymptotically stable when $R_0 < 1$ [41]. So, if the drug epidemic in a population is controlled to a point such that it is sufficiently close to the DFE, then a drug-free state may be reached. Ultimately, the authors conclude that minimizing the number of individuals who become addicted to drugs is more useful to preventing an epidemic than increasing the number of people with an addiction who are accessing treatment. White and Comiskey's (2007) paper helps to specify the best goals for ending an opioid epidemic, with extensions of their work reaching well beyond the original scope.

Battista et al. (2019) is one such paper that extends White & Comiskey's (2007) work. Battista et al. (2019) uses an SIR-type model for addiction to a general class of prescription drugs with 4 compartments: susceptible (S) individuals who are not using opioids or actively recovering from

addiction, prescribed users (P) who do not have an addiction to their prescribed drugs, addicted (A) individuals who are addicted to opioids, whether they are prescribed or not, and recovered (R) individuals who are in treatment for their opioid addiction. Based on the opioid crisis in the United States, Battista et al. (2019) seek to describe the best national strategies that will eventually bring the epidemic to an end. One advantage of the model is that it focuses on addiction which arises from prescribed medicines, rather than only illicit drug use. This has significant implications regarding the understanding of, and future legislation pertaining to prescription drugs. The main assumption of the paper is that the sum of the compartments is 1; that is, every individual in the population is in exactly one compartment. Additionally, the authors assume that the population size is constant, that opioid-overdose related deaths do not significantly change total population proportions, the primary source of opioids for most users is prescribed medication, and that relapse is an inherent part of the drug using career [2].

The R_0 of this model is derived, under the assumptions that individuals do not become addicted to their own prescription and that new prescriptions do not increase addiction to illicit substances, to be

$$R_0 = \frac{\beta_A S^*}{\mu^* + \zeta\Lambda}, \text{where } \Lambda = \frac{\delta+\mu}{\delta+\mu+\sigma} \text{ and } S^* = \frac{\epsilon+\mu}{\alpha+\epsilon+\mu}$$

This emphasizes the idea that the predominant cause of addiction in the system is over-prescription by medical professionals which allows for diversion and thus abuse. Battista et al. (2019) then consider the endemic equilibrium (non-DFE), where there are persistent but constant rates of addiction in the population in the long run. An equilibrium is found to exist when all parameters are non-negative and the rate of illicit addiction base on the A class is positive. The authors find that if both a DFE and an endemic equilibrim exist, then the DFE is only reached when it is locally stable. A sensitivity analysis of R_0 concludes that sufficient focus on the prevention of prescription-induced addiction is a critical step in combating the crisis and may contain the epidemic to relatively low endemic states [2]. Ultimately, Battista et al. (2019) provide a foundational understanding of the opioid crisis and the potential success of intervention actions such as decreasing prescriptions, increasing treatment success rates, and increasing the availability and entrance into treatment. Their research leads to further extension that considers the role of non-prescription opioids such as heroin and fentanyl that have become more prominent in recent years.

2 Motivation

The goal of the following work is to develop a deeper understanding of the opioid crisis in the United States, with a particular focus on the effectiveness of the HELP4WV call center in increasing the proportion of West Virginia drug users who recover with an active mindset, and thus ultimately increasing long-term recovery from opioid addiction in the system. We define a drug user as an individual who has been diagnosed with an OUD. This model is unique in that we use a system of differential equations to mimic the drug using career by quantifying qualitative recovery mindsets. This section highlights the differences between active and passive recovery and discusses why and how we quantify this information in order to build our model.

According to Windmill Wellness Ranch [42], a passive approach to recovery is one in which an individual may outwardly appear to be recovering, yet they hold a negative mindset regarding the reality of their ability to manage their SUD. An individual with a passive mindset may believe that long-term sobriety is possible alone, by going to enough narcotics anonymous meetings, building enough habits that supposedly lead to sober living, abstinence, a combination of each, etc., or they may refuse to believe at all in the real possibility of long-term recovery. Alternatively, an active approach to recovery is one in which an individual recognizes and accepts that recovery is hardly attainable alone. Instead, it is vital for their health that they accept help from others and truly believe in their ability to manage their SUD in a healthy manner. These categories are clearly qualitatively differentiated; we need quantifiable information to build a useful model and draw meaningful conclusions. So, with the goal of determining if an active response is more likely to lead to long term recovery compared to a passive response, we seek a quantifiable method to discern the difference between the two and thus analyze the effectiveness of call centers in controlling the opioid epidemic.

HELP4WV is a call center located in West Virginia that operates 24/7 to connect individuals with in-state health resources, especially regarding addiction and mental health issues. After years of comprehensive assessment of the state's needs and other development research, the Department of Health and Human Resources (HHS) funded organization launched in 2015 with the principal goal to act as a "centralized hub of information resources available" [26]. This business model was adopted as the aforementioned needs assessment showed that, because they did not know where to look for help, many West Virginians felt that addiction treatment was not available; these individuals were therefore unable to access the assistance they required [26]. Another key aspect of HELP4WV's model is that the call center initiates follow-up calls with every individual at the 48-hour, 1-week, and 1-month checkpoints to ensure that recovery efforts remain on track. These calls allow individuals to feel important by realizing that they are not just another voice on the other end of a line - a real person from the call center reaches back out to them at least 3 times within one month of the initial call for help. During these calls, the agent is able to share relevant resources and design a solution-focused plan for the caller's specific wants and needs that may include setting up doctor appointments, connecting the caller to community members, and more [26]. Consequently, every caller is able to access personalized care and support for their recovery, regardless of their current drug using, geographic, financial, or health situation.

HELP4WV's business model is clearly an example of advocacy for active recovery- the call center gives individuals the opportunity and resources necessary to receive help and begin to self-believe in their ability to recover. We thus use call data from the center to define active and passive recovery mindsets as such: if an individual answers all three follow-up calls, we consider them to have an active mindset regarding their recovery; if an individual fails to answer one or more of the calls, we say that the hesitation to stay connected with help indicates that they are using a passive mindset. This application of epidemiological modeling to the drug using problem is novel because it uses data from a particular method of intervention - the HELP4WV call center - to study the impact of an individual's mindset as they begin or attempt to recover from an OUD.

The remainder of the paper is structured as follows. Chapter 3 discusses the formulation of the model, including an overview of the assumptions and parameterization. In Chapter 4 we analyze

the model behavior to derive the basic reproduction number R_0, analyze the system stability when $R_0 < 1$, and discuss any endemic equilibrium solutions. Finally, Chapter 5 will provide a discussion of our findings and possible directions for future study.

3 Model

The population of West Virginians is split into five mutually exclusive compartments. First, the susceptible (S) population is those West Virginians who have never developed an OUD. Individuals considered using (U_1) have developed an OUD but have either never called HELP4WV or have called before but relapsed after using a passive mindset. Individuals recovering with an active mindset (A) are characterized by answering all 3 follow-up calls, while the passive mindset group (P) are those who have called HELP4WV but failed to answer at least follow-up call. Lastly, recovered (R) individuals are those who have remained sober for at least 90 consecutive days starting at their initial outreach call to the center. Individuals who have recovered may have done so using either an active or passive mindset. These compartments were defined based on White and Comiskey's (2007) model. Our infectious group, U_1, retains the inclusion of both first-time addiction development and addiction from relapse. We then split White and Comiskey's (2007) U_2 compartment into two groups (A and P) in order to distinguish individuals recovering with an active versus passive mindset. Finally, we add a recovered (R) compartment to describe those who have remained sober for 3 months.

A few limitations of our model exist due to parameters that were omitted to simplify model analysis. For one, relapse is permitted from the passive compartment, but not for those individuals who use an active mindset or who have attained at least 90 days of sobriety. Obviously it is possible to relapse at any point in one's recovery journey, even with a strong support system and sense of self-belief. Additionally, our model only considers recovery through HELP4WV; individuals may not recover by other means. This is because the call center is not a treatment itself, but rather connects callers to a treatment resource. Additionally, we are specifically interested in the effectiveness of HELP4WV on helping to curb the opioid crisis in West Virginia. Though allowing relapse from other stages in the drug-using career or recovery by means other than using the call center may better represent the real-world situation, we leave those model dynamics to future study in order to build a model from which we can draw meaningful conclusions.

A final limitation is that data used in this model are specific to West Virginia, which has a higher rate of opioid addiction than the national average. This means that the model may be difficult to generalize to a wider scope. However, some parametric values for this model come from national statistics and thus result in a conservative model of the opioid crisis situation in West Virginia.

The model diagram is shown in Figure 2 below.

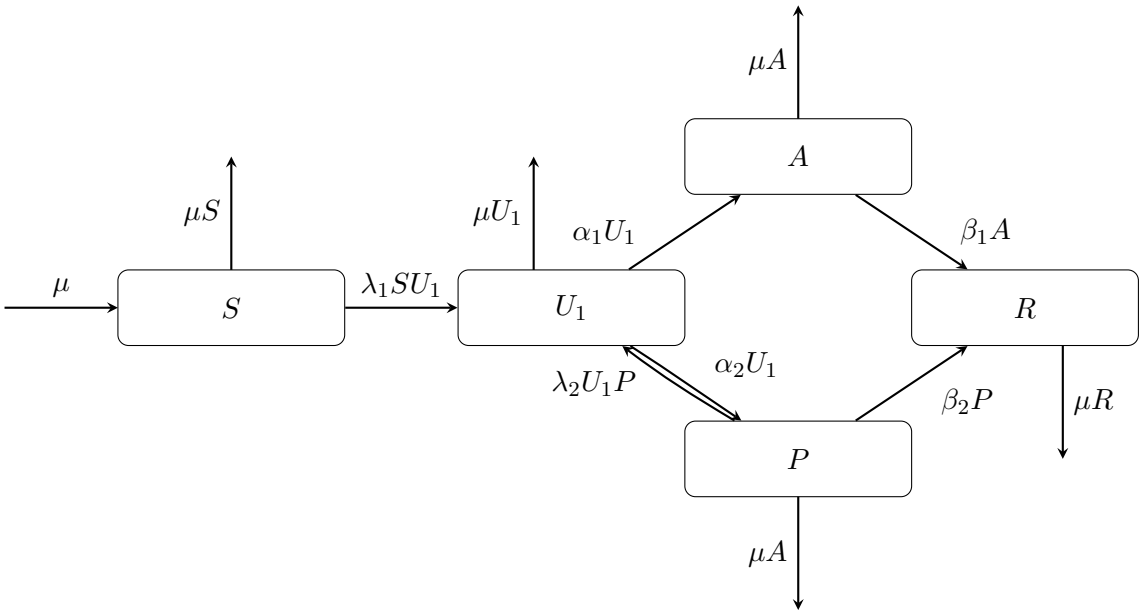

Figure 2: Diagram of our model

The basic assumption of our model is that the drug using population can be compartmentalized into five distinct groups. The other assumptions that our model relies on are as follows:

(1) The population size remained constant during the modeling period

(2) The population mixed homogeneously

(3) All follow-up calls made at the 1-month checkpoint were originally SUD related

(4) If an individual does not answer the follow-up call at one checkpoint then the center will not call them at the next checkpoint

Assumptions (1) and (2) follow from typical SIR-type model assumptions [34], [3]. We can assume (3) because initial calls that were not about a SUD were predominantly individuals seeking either specific information or emotional support - neither of which would warrant a 1-month follow up call initiated by HELP4WV [18]. This allows us to draw meaningful conclusions from the number of answered follow-up calls, since we can compare the number of answered calls to the number of initial calls regarding SUD. Finally, we are able to assume (4) since the center reaches out 3 times at each checkpoint [26]. A missed call then implies that the person actively did not want to respond rather than they simply were busy at the time. This again increases the significance of the meaning of answered follow-up calls. An individual who answers the one-month call under this assumption has been in consistent contact with the call center and displays earnest, active participation in their recovery.

The model's system of equations is given below, followed by a table of parameters, defined and valued, in Table 1.

$$
\begin{aligned}
\frac{dS}{dt} &= \mu - \mu S - \lambda_1 S U_1 \\
\frac{dU_1}{dt} &= \lambda_1 S U_1 - \mu U_1 - \alpha_1 U_1 - \alpha_2 U_1 + \lambda_2 U_1 P \\
\frac{dA}{dt} &= \alpha_1 U_1 - \mu A - \beta_1 A \\
\frac{dP}{dt} &= \alpha_2 U_1 - \mu P - \beta_2 P - \lambda_2 P U_1 \\
\frac{dR}{dt} &= \beta_1 A + \beta_2 P - \mu R
\end{aligned} \tag{1}
$$

Parameter	Description	Value	Source
μ	death rate	0.00728	CDC [24]
λ_1	probability of developing an OUD due to interacting with an individual with an OUD	.36	Han et al. [17], Hughes et al. [19]
α_1	proportion of callers who answer all 3 follow-up calls	0.06981	HELP4WV [26], [18], NSDUH [13]
α_2	proportion of callers who fail to answer one or more follow-up calls	0.18144	HELP4WV [26], [18], NSDUH [13]
β_1	recovery rate from A; individual is sober for 90 consecutive days starting from their initial outreach to the center	0.00455	NSDUH [13], Weiss [38], Weiss & Rao [39]
β_2	recovery rate from P; individual is sober for 90 consecutive days starting from their initial outreach to the center	0.00152	NSDUH [13], Weiss [38], Weiss & Rao [39]
λ_2	relapse rate from P; relapse within 1 month of calling the center	0.65	Broers et al. [5]

Table 1: Model parameters

The parameter values given were chosen based on the literature review conducted. Specifically, α_1 and α_2 were found by estimating the number of answered 1-month calls, given the initial call was regarding an OUD. In 2021, HELP4WV took 12,458 total calls; of these, 42% were related to a SUD [18]. According to the 2020 National Survey on Drug Use and Health [13], nearly 7% of individuals with a SUD struggle with opioids specifically. So, we can estimate that 12,458(.42)(.069) = 3610 calls to HELP4WV were about an OUD in 2021. Using the most recent data available, 504 individuals answered their 1-month follow call in 2016 [26]. Assuming that 50% of 1-month calls were related to an OUD, we can calculate α_1, the probability that someone answers the 1-month

call, given that they called the center and their initial outreach was regarding an OUD. Then α_2 is calculated analogously but considering those who were called at the 1-month mark but did not answer.

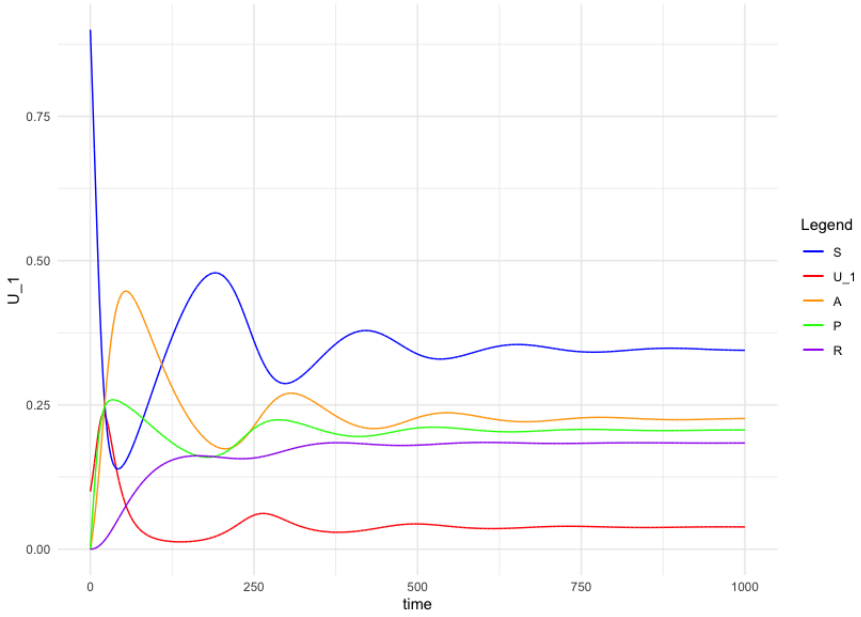

Figure 3: System dynamics over time
Parameters: refer to Table 1

The graph of the system with these parameters, displayed in Figure 3, shows that there exists an endemic equilibrium in the long run. There will eventually be no net change in each compartment, but a positive proportion of individuals with an OUD who are thus infectious to the population. So, HELP4WV alone does not appear to be enough to contain the opioid epidemic in West Virginia.

4 Methods

An analysis of the model begins with defining the DFE of the system. We then use the next generation method to derive the R_0 value for the model and use the Jacobian matrix to discuss the stability of the DFE when $R_0 < 1$. Next we consider the possible existence of equilibrium solutions when $R_0 \geq 1$.

4.1 The basic reproduction number

The DFE is found when the population is wholly susceptible. That is, $(S, U_1, A, P, R) = (1, 0, 0, 0, 0)$. Using the next generation method [6], we find

$$\mathcal{F} = [\lambda_1 S + \lambda_2 P U_1] \text{ and}$$

$$\mathcal{V} = [\alpha_1 U_1 + \alpha_2 U_1 + \mu - \lambda_2 P U_1],$$

where $\mathcal{F}$ represents the influx of newly infected individuals into the U_1 compartment and $\mathcal{V}$ represents any other transfers related to the infectious compartment. Because those in A and P are in treatment, we do not consider them infectious to the population in our model. These matrices yield

$$F = [\ \lambda_1] \text{ and}$$
$$V = [\ \alpha_1 + \alpha_2 + \mu]$$

respectively when evaluated at the DFE. It is important to mention here that we define "new infections" as both individuals developing an OUD for the first time as well as those relapsing from recovery with a passive mindset. This follows Driessche & Watmough's (2008) example of a disease treatment model that includes reinfection - a distorted Castillo-Chavez & Feng (1997) model for tuberculosis [16], [7]. For models that include re-infection, it is generally a better epidemiological interpretation to consider reinfection as new [16].

However, whether relapse is considered a new addiction or not in our model does not ultimately effect the R_0 value, as $\mathcal{F}$ and $\mathcal{V}$ are evaluated at the DFE. The next generation matrix $K = FV^{-1}$ in either case is then

$$K = [\tfrac{\lambda_1}{\alpha_1+\alpha_2+\mu}],$$

whose singular entry is also the spectral radius ρ of the matrix; $R_0 = \frac{\lambda_1}{\alpha_1+\alpha_2+\mu}$ for the system [6], [15].

4.2 Stability analysis of the DFE when $\mathbf{R_0 < 1}$

So long as the eigenvalues of the Jacobian matrix of the model, when evaluated at the DFE, have real, negative parts, the DFE of an epidemiological model is known to be locally asymptotically stable when $R_0 < 1$ [14].

The Jacobian of the system is a matrix of first-order partial derivatives:

$$J(S, U_1, A, P, R) = \begin{bmatrix} -\lambda_1 U_1 - \mu & -\lambda_1 S & 0 & 0 & 0 \\ \lambda_1 U_1 & \lambda_1 S - \mu - \alpha_1 - \alpha_2 + \lambda_2 P & 0 & \lambda_2 U_1 & 0 \\ 0 & \alpha_1 & -\mu - \beta_1 & 0 & 0 \\ 0 & \alpha_2 - \lambda_2 P & 0 & -\mu - \beta_2 - \lambda_2 U_1 & 0 \\ 0 & 0 & \beta_1 & \beta_2 & -\mu \end{bmatrix}$$

Evaluated at the DFE,

$$J^*(1, 0, 0, 0, 0) = \begin{bmatrix} -\mu & -\lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_1 - \mu - \alpha_1 - \alpha_2 & 0 & 0 & 0 \\ 0 & \alpha_1 & -\mu - \beta_1 & 0 & 0 \\ 0 & \alpha_2 & 0 & -\mu - \beta_2 & 0 \\ 0 & 0 & \beta_1 & \beta_2 & -\mu \end{bmatrix}$$

The eigenvalues of this matrix are as follows:

$$
\begin{aligned}
eig_1 &= -\mu \\
eig_2 &= -\mu - \beta_1 \\
eig_3 &= -\mu - \beta_2 \\
eig_4 &= -\mu \\
eig_5 &= \lambda_1 - \alpha_1 - \alpha_2 - \mu
\end{aligned}
$$

Clearly the first 4 eigenvalues satisfy the real and negative conditions. Given $R_0 < 1$ is true, then eig_5 is real and negative as well, and the DFE is locally asymptotically stable. There are 3 ways to decrease R_0 such that it is sufficiently low; it is obviously unethical to increase the death rate μ. If the effective contact rate λ_1 between susceptible individuals and those with an OUD is sufficiently decreased, the epidemic may be supressed. Assuming λ_1 remains large, HELP4WV must significantly increase their reach if the opioid crisis is to be contained.

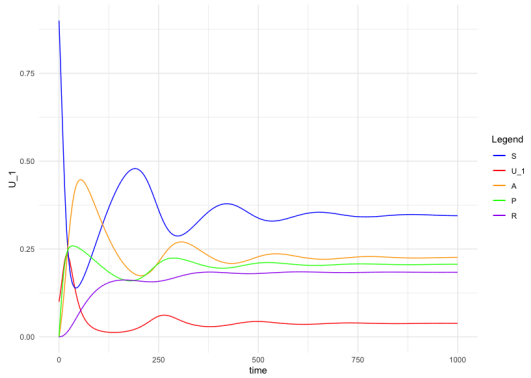

Figure 4: $R_0 > 1$ (Endemic Equilibrium)
$\mu = .00728, \lambda_1 = .5,$
$\alpha_1 = .06981, \alpha_2 = .18144,$
$\beta_1 = .00455, \lambda_2 = .65, \beta_2 = .00152$

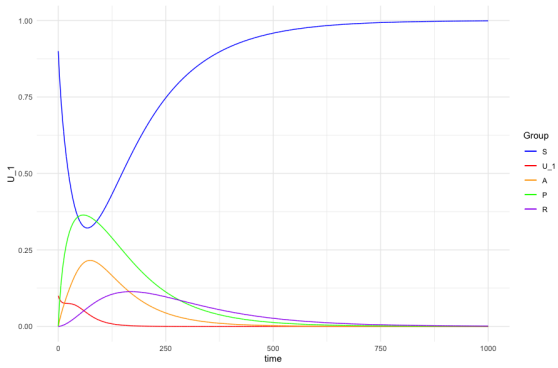

Figure 5: $R_0 < 1$ (Disease Free Equilibrium)
$\mu = .00728, \lambda_1 = .5,$
$\alpha_1 = .2985, \alpha_2 = .18144$
$\beta_1 = .00455, \lambda_2 = .65, \beta_2 = .00152$

The graphs above show that even with a high rate $\lambda_1 = .5$ of OUD development, a sufficient increase in the proportion α_1 of individuals who recover with an assertive mindset will make $R_0 < 1$ become true and thus suppress the opioid crisis. It is clear that a sufficient increase in α_2 would similarly result in a DFE. Though it may seem counter-intuitive that increasing the proportion of individuals using a passive mindset may contain the opioid epidemic, consider the role of HELP4WV. Even if an individual fails to answer one or more follow-up calls, they have become connected to resources they did not have access to before making their initial call to the center. Either that treatment was able to help them enough to reach recovery, or they may relapse. If an individual relapses, they may continue to use drugs without attempting to recover. Alternatively, they may call HELP4WV again. Given their past experience, and that they are reaching out to the center for a subsequent time, an individual may use an active mindset instead of a passive mindset to

approach their recovery journey. Ultimately, HELP4WV connects drug users to resources regardless of the mindset of individual callers. That is why increasing α_1 or α_2 may both help reduce R_0 enough to control the epidemic.

4.3 Endemic equilibrium solution(s)

We now obtain non-trivial equilibrium values for each of the variables S, U_1, A, P, R such that each variable has a non-negative value. By definition, each equation in system (1) is thus set to zero. Solving the first equation in system (1) as such, we see

$$\begin{aligned} 0 &= \mu - \lambda_1 S U_1 - \mu S \\ 0 &= \mu - S(\lambda_1 U_1 + \mu) \\ S(\lambda_1 U_1 + \mu) &= \mu \\ S^* &= \frac{\mu}{\lambda_1 U_1 + \mu} \end{aligned}$$

Solving the second equation from system (1):

$$\begin{aligned} 0 &= \lambda_1 S U_1 - \mu U_1 - \alpha_1 U_1 - \alpha_2 U_1 + \lambda_2 U_1 P \\ &= (\lambda_1 S - \mu - \alpha_1 - \alpha_2 + \lambda_2 P) U_1 \end{aligned}$$

If $\frac{dU_1}{dt} = 0$, then either $U_1 = 0$ or $(\lambda_1 S - \mu - \alpha_1 - \alpha_2 + \lambda_2 P) = 0$. As the former is a trivial condition, we focus on the latter and, using the value for S just found, see that

$$\begin{aligned} 0 &= \lambda_1 S - \mu - \alpha_1 - \alpha_2 + \lambda_2 P \\ 0 &= \lambda_1 \left(\frac{\mu}{\lambda_1 U_1 + \mu} \right) - \mu - \alpha_1 - \alpha_2 + \lambda_2 P \\ \mu + \alpha_1 + \alpha_2 - \lambda_1 \left(\frac{\mu}{\lambda_1 U_1 + \mu} \right) &= \lambda_2 P \\ P^* &= \frac{1}{\lambda_2} \left(\mu + \alpha_1 + \alpha_2 - \lambda_1 \left(\frac{\mu}{\lambda_1 U_1 + \mu} \right) \right) \end{aligned}$$

The third equation from system (1) yields:

$$\begin{aligned} 0 &= \alpha_1 U_1 - \mu A - \beta_1 A \\ A(\mu + \beta_1) &= \alpha_1 U_1 \\ A^* &= \frac{\alpha_1 U_1}{\mu + \beta_1} \end{aligned}$$

Finally, we find R^*:

$$\begin{aligned}
0 &= \beta_1 A + \beta_2 P - \mu R \\
\mu R &= \beta_1 A + \beta_2 P \\
R^* &= \frac{\beta_1 A + \beta_2 P}{\mu} \\
R^* &= \frac{\beta_1}{\mu}\left(\frac{\alpha_1 U_1}{\mu+\beta_1}\right) + \frac{\beta_2}{\mu\lambda_2}\left(\mu + \alpha_1 + \alpha_2 - \lambda_1\left(\frac{\mu}{\lambda_1 U_1 + \mu}\right)\right)
\end{aligned}$$

Using $(S^*, U_1^*, A^*, P^*, R^*)$ as found above, we can now use our basic assumption to solve $S^* + U_1^* + A^* + P^* + R^* = 1$, evaluated at the non-trivial equilibrium:

$$\frac{\mu}{\lambda_1 U_1^* + \mu} + U_1^* + \frac{\alpha_1 U_1^*}{\mu+\beta_1} + \frac{1}{\lambda_2}\left[\mu + \alpha_1 + \alpha_2 - \lambda_1\left(\frac{\mu}{\lambda_1 U_1^* + \mu}\right)\right] + \frac{\beta_1}{\mu}\left(\frac{\alpha_1 U_1}{\mu+\beta_1}\right) + \frac{\beta_2}{\mu\lambda_2}\left[\mu + \alpha_1 + \alpha_2 - \lambda_1\left(\frac{\mu}{\lambda_1 U_1 + \mu}\right)\right] = 1$$

$$\frac{\mu}{\lambda_1 U_1^* + \mu} + \frac{\alpha_1 U_1^*}{\mu+\beta_1} + \frac{\beta_1}{\mu}\left(\frac{\alpha_1 U_1}{\mu+\beta_1}\right) + \frac{\beta_2}{\mu\lambda_2}\left[\mu + \alpha_1 + \alpha_2 - \lambda_1\left(\frac{\mu}{\lambda_1 U_1 + \mu}\right)\right] + \frac{1}{\lambda_2}\left[\mu + \alpha_1 + \alpha_2 - \lambda_1\left(\frac{\mu}{\lambda_1 U_1^* + \mu}\right)\right] + U_1^* = 1$$

$$\frac{\mu}{\lambda_1 U_1^* + \mu} + \frac{\alpha_1 U_1^*}{\mu+\beta_1} + \frac{\beta_1}{\mu}\left(\frac{\alpha_1 U_1}{\mu+\beta_1}\right) + \frac{\beta_2}{\mu\lambda_2}(\mu + \alpha_1 + \alpha_2) - \frac{\beta_2\lambda_1}{\mu\lambda_2}\left(\frac{\mu}{\lambda_1 U_1^* + \mu}\right) + \frac{1}{\lambda_2}\left[\mu + \alpha_1 + \alpha_2 - \lambda_1\left(\frac{\mu}{\lambda_1 U_1^* + \mu}\right)\right] + U_1^* = 1$$

$$\frac{\alpha_1 U_1^*}{\mu+\beta_1}\left(1 + \frac{\beta_1}{\mu}\right) + \frac{\beta_2}{\mu\lambda_2}(\mu + \alpha_1 + \alpha_2) + \frac{\mu}{\lambda_1 U_1^* + \mu}\left(1 - \frac{\beta_2\lambda_1}{\mu\lambda_2}\right) + \frac{1}{\lambda_2}\left[\mu + \alpha_1 + \alpha_2 - \lambda_1\left(\frac{\mu}{\lambda_1 U_1^* + \mu}\right)\right] + U_1^* = 1$$

$$\left[\frac{\alpha_1 U_1^*}{\mu+\beta_1}\left(1 + \frac{\beta_1}{\mu}\right) + 1\right] U_1^* + \frac{\mu}{\lambda_1 U_1^* + \mu}\left(1 - \frac{\beta_2\lambda_1}{\mu\lambda_2} - \frac{\lambda_1}{\lambda_2}\right) + (\mu + \alpha_1 + \alpha_2)\left(\frac{\beta_2}{\mu\lambda_2} + \frac{1}{\lambda_2}\right) - 1 = 0$$

$$\left(\frac{\alpha_1\mu}{(\mu+\beta_1)\mu} + \frac{\alpha_1\mu\beta_1}{(\mu+\beta_1)\mu} + 1\right) U_1^* + \frac{\mu}{\lambda_1 U_1^* + \mu}\left(1 - \frac{\beta_2\lambda_1}{\mu\lambda_2} - \frac{\lambda_1}{\lambda_2}\right) + (\mu + \alpha_1 + \alpha_2)\left(\frac{\beta_2}{\mu\lambda_2} + \frac{1}{\lambda_2}\right) - 1 = 0$$

$$\left(\frac{\alpha_1(\mu+\beta_1)}{(\mu+\beta_1)\mu} + 1\right) U_1^* + \frac{\mu}{\lambda_1 U_1^* + \mu}\left(1 - \frac{\beta_2\lambda_1}{\mu\lambda_2} - \frac{\lambda_1}{\lambda_2}\right) + (\mu + \alpha_1 + \alpha_2)\left(\frac{\beta_2}{\mu\lambda_2} + \frac{1}{\lambda_2}\right) - 1 = 0$$

$$\left(\frac{\alpha_1}{\mu} + 1\right) U_1^* + \frac{\mu}{\lambda_1 U_1^* + \mu}\left(1 - \frac{\beta_2\lambda_1}{\mu\lambda_2} - \frac{\lambda_1}{\lambda_2}\right) + (\mu + \alpha_1 + \alpha_2)\left(\frac{\beta_2}{\mu\lambda_2} + \frac{1}{\lambda_2}\right) - 1 = 0$$

If

$$\frac{\mu}{\lambda_1 U_1^* + \mu}\left(1 - \frac{\beta_2\lambda_1}{\mu\lambda_2} - \frac{\lambda_1}{\lambda_2}\right) = 0$$

is true, then either

$$\begin{aligned} \mu &= 0 \\ 1 - \frac{\beta_2 \lambda_1}{\mu \lambda_2} - \frac{\lambda_1}{\lambda_2} &= 0 \end{aligned}$$

or both are true. Obviously the population death rate cannot be zero, so we turn to the second condition.

$$\begin{aligned} 1 - \frac{\beta_2 \lambda_1}{\mu \lambda_2} - \frac{\lambda_1}{\lambda_2} &= 0 \\ \frac{\beta_2 \lambda_1}{\mu \lambda_2} + \frac{\lambda_1}{\lambda_2} &= 1 \\ \lambda_1 \left(\frac{\beta_2}{\mu \lambda_2} + \frac{1}{\lambda_2} \right) &= 1 \\ \frac{\beta_2}{\mu \lambda_2} + \frac{1}{\lambda_2} &= \frac{1}{\lambda_1} \end{aligned}$$

This can be used to solve the following:

$$\left(\frac{\alpha_1}{\mu} + 1 \right) U_1^* + (\mu + \alpha_1 + \alpha_2) \left(\frac{\beta_2}{\mu \lambda_2} + \frac{1}{\lambda_2} \right) - 1 = 0$$

$$\left(\frac{\alpha_1}{\mu} + 1 \right) U_1^* + (\mu + \alpha_1 + \alpha_2) \left(\frac{1}{\lambda_1} \right) - 1 = 0$$

$$\left(\frac{\alpha_1}{\mu} + 1 \right) U_1^* = 1 - (\mu + \alpha_1 + \alpha_2) \left(\frac{1}{\lambda_1} \right)$$

$$U_1^* = \frac{1 - (\mu + \alpha_1 + \alpha_2) \left(\frac{1}{\lambda_1} \right)}{\frac{\alpha_1}{\mu} + 1}$$

$$\begin{aligned} U_1^* &= \frac{\mu}{\alpha_1 + \mu} \left(1 - (\mu + \alpha_1 + \alpha_2) \left(\frac{1}{\lambda_1} \right) \right) \\ &= \frac{\mu}{\alpha_1 + \mu} \left(1 - \frac{1}{R_0} \right) \end{aligned}$$

So long as this value is positive, there exists an endemic equilibrium solution for the system. That is, if $R_0 > 1$ is true, then the system will always include a positive proportion of individuals with an OUD in the long run. If the supposition that

$$\frac{\mu}{\lambda_1 U_1^* + \mu} \left(1 - \frac{\beta_2 \lambda_1}{\mu \lambda_2} - \frac{\lambda_1}{\lambda_2} \right) = 0$$

is not true, then we return to the previous line

$$\left(\frac{\alpha_1}{\mu}+1\right)U_1^* \;+\; \frac{\mu}{\lambda_1 U_1^*+\mu}\left(1-\frac{\beta_2\lambda_1}{\mu\lambda_2}-\frac{\lambda_1}{\lambda_2}\right) \;+\; (\mu \;+\; \alpha_1 \;+\; \alpha_2)\left(\frac{\beta_2}{\mu\lambda_2}+\frac{1}{\lambda_2}\right) \;-\; 1 \quad = \quad 0$$

and continue to solve for U_1^*. In order to simplify computation, we define the following:

$$\begin{aligned}
X &= \frac{\alpha_1}{\mu}+1, \\
Y &= 1-\frac{\beta_2\lambda_1}{\mu\lambda_2}-\frac{\lambda_1}{\lambda_2} \text{ and} \\
Z &= (\mu+\alpha_1+\alpha_2)\left(\frac{\beta_2}{\mu\lambda_2}+\frac{1}{\lambda_2}\right)-1
\end{aligned}$$

and thus continue:

$$\begin{aligned}
0 &= XU_1^*+\frac{\mu}{\lambda_1 U_1^*+\mu}Y+Z \\
&= (\lambda_1 U_1^*+\mu)XU_1^*+\mu Y+(\lambda_1 U_1^*+\mu)Z \\
&= \lambda_1 XU_1^{*2}+\mu\lambda_1 U_1^*+\mu Y+\lambda_1 ZU_1^*+\mu Z \\
&= \lambda_1 XU_1^{*2}+(\mu X+\lambda_1 Z)U_1^*+\mu(Y+Z) \\
&= \lambda_1\left(\frac{\alpha_1}{\mu}+1\right)U_1^{*2}+\left[\mu\left(\frac{\alpha_1}{\mu}+1\right)+\lambda_1\left((\mu+\alpha_1+\alpha_2)\left(\frac{\beta_2}{\mu\lambda_2}+\frac{1}{\lambda_2}\right)-1\right)\right]U_1^*+ \\
&\mu\left(\left[1-\frac{\beta_2\lambda_1}{\mu\lambda_2}-\frac{\lambda_1}{\lambda_2}\right]+\left[(\mu+\alpha_1+\alpha_2)\left(\frac{\beta_2}{\mu\lambda_2}+\frac{1}{\lambda_2}\right)-1\right]\right) \\
&= \lambda_1\left(\frac{\alpha_1}{\mu}+1\right)U_1^{*2}+\left[\alpha_1+\mu+\frac{\mu+\alpha_1+\alpha_2}{\lambda_2}\left(\frac{\beta_2}{\mu}+1\right)\lambda_1-\lambda_1\right]U_1^*+ \\
&\mu-\frac{\beta_2\lambda_1}{\lambda_2}-\frac{\lambda_1\mu}{\lambda_2}+\left(\frac{\mu+\alpha_1+\alpha_2}{\lambda_2}\right)(\beta_2+\mu)-\mu \\
&= \lambda_1\left(\frac{\alpha_1}{\mu}+1\right)U_1^{*2}+\left[\alpha_1+\mu+\left(\frac{\mu+\alpha_1+\alpha_2}{\lambda_2}\left(\frac{\beta_2+\mu}{\mu}\right)-1\right)\lambda_1\right]U_1^*+ \\
&-\frac{\lambda_1}{\lambda_2}(\beta_2+\mu)+(\frac{\mu+\alpha_1+\alpha_2}{\lambda_2})(\beta_2+\mu) \\
&= \lambda_1\left(\frac{\alpha_1}{\mu}+1\right)U_1^{*2}+\left[\alpha_1+\mu+\left(\frac{\mu+\alpha_1+\alpha_2}{\lambda_2}\left(\frac{\beta_2+\mu}{\mu}\right)-1\right)\lambda_1\right]U_1^*+ \\
&\left(\frac{\mu+\alpha_1+\alpha_2}{\lambda_2}\right)(\beta_2+\mu)-\frac{\lambda_1}{\lambda_2}(\beta_2+\mu)
\end{aligned}$$

$$= \lambda_1 \left(\frac{\alpha_1}{\mu} + 1 \right) U_1^{*2} + \left[\alpha_1 + \mu + \left(\frac{\mu + \alpha_1 + \alpha_2}{\lambda_2} \left(\frac{\beta_2 + \mu}{\mu} \right) - 1 \right) \lambda_1 \right] U_1^* +$$
$$(\mu + \alpha_1 + \alpha_2 - \lambda_1) \left(\frac{\beta_2 + \mu}{\lambda_2} \right)$$
$$= \lambda_1 \left(\frac{\alpha_1}{\mu} + 1 \right) U_1^{*2} + \left[\alpha_1 + \mu + \left(\frac{\mu + \alpha_1 + \alpha_2}{\lambda_2} \left(\frac{\beta_2 + \mu}{\mu} \right) - 1 \right) \lambda_1 \right] U_1^* +$$
$$\left(\frac{\beta_2 + \mu}{\lambda_2} \right) (\mu + \alpha_1 + \alpha_2)(1 - R_0)$$

We now have a quadratic equation for U_1^* in the form $AU_1^{*2} + BU_1^* + C = 0$, where

$$A = \lambda_1 \left(\frac{\alpha_1}{\mu} + 1 \right),$$
$$B = \alpha_1 + \mu + \left(\frac{\mu + \alpha_1 + \alpha_2}{\lambda_2} \left(\frac{\beta_2 + \mu}{\mu}] \right) - 1 \right) \lambda_1 \qquad \text{and}$$
$$C = \left(\frac{\beta_2 + \mu}{\lambda_2} \right) (\mu + \alpha_1 + \alpha_2)(1 - R_0)$$

Using Descarte's Rule of Signs [33], U_1 must be real and positive for an endemic equilibrium to exist. Clearly, A is always real and positive. As for B and C, we investigate the existence of an endemic equilibrium for 3 possible cases:

Case 1: $\mathbf{R_0 > 1}$ and $\mathbf{B < 0}$. In this scenario, because $R_0 > 1 \implies C < 0$, there is one sign change. Therefore, there is always at least one real, positive solution for U_1^* and thus an endemic equilibrium always exists under these conditions.

Case 2: $\mathbf{R_0 > 1}$ and $\mathbf{B > 0}$. Similar to case 1, this case yields a negative value for C and therefore one sign change. There is always at least one real, positive solution for U_1^* and thus an endemic equilibrium always exists under these conditions.

Case 3: $\mathbf{R_0 = 1}$. In this case, $R_0 = 1 \implies C = 0$, and the quadratic is reduced to $AU_1^{*2} + BU_1^* = 0$; this can be rewritten as $U_1^*(AU_1^* + B) = 0$ and then easily solved. The equation holds true if $U_1^* = 0, \frac{-B}{A}$, the former of which is a trivial DFE. So in order for U_1^* to have a real, positive value and therefore an endemic solution to exist, the coefficient B of the quadratic equation must be negative.

So, there always exists an endemic equilibrium when $R_0 > 1$ is true and if $R_0 = 1$ then an endemic equilibrium may exist under specific parametric conditions.

5 Discussion

In this paper we present a model that studies the effectiveness of call centers on the opioid epidemic by quantitatively differentiating recovery from addiction using an active mindset versus recovery using a passive mindset. A key result of our model analysis is the basic reproduction number $R_0 = \frac{\lambda_1}{\mu + \alpha_1 + \alpha_2}$. This threshold is interpreted as follows: in order to control the opioid epidemic, it is necessary that the rate of developing an OUD due to the interaction of susceptible individuals

and drug users be less than the sum of the rates of cessation of addiction. When $R_0 < 1$ is true, the system is locally asymptotically stable and an DFE may be reached. When $R_0 > 1$, there exists an endemic equilibrium, and when $R_0 = 1$ a DFE equilibrium may always exist and an endemic equilibrium exists for particular initial parametric states.

While our parametric values rely on data specific to the state of West Virginia, the model built has opportunity to be expanded beyond the original scope of this paper. Our findings provide a mathematical basis on which to continue research on the opioid epidemic in order to better understand successful intervention methods to control opioid abuse, and more importantly, deaths due to opioid overdose. Further analysis of the model can provide crucial information to consider for possible policy decisions. For example, a sensitivity analysis on R_0 can more explicitly describe the impact of specific parameters on the long-term behavior of the system with a given initial state. In particular, it is of great interest whether changes in the rate of recovery with an active mindset or the rate of recovery with a passive mindset more significantly effects the proportion of individuals who eventually see long-term recovery.

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