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Entropy in Music: An Analysis of Shape Note Music in Terms of Information Entropy

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1. Abstract

At first glance, the disciplines of music and computer science might seem like distinct and almost mutually exclusive fields. Music is often thought of as a subjective discipline rooted largely in a complex balance of aesthetic qualities such as pitch, rhythm, intonation, and dynamic contrast, while computer science is often seen as a discipline grounded in the ability to work with, and sometimes even think like calculating, logical, efficient machinery. And yet beneath the surface there are plenty of ways in which human sensibilities become important in computer science. One major example would be the entire subfield of Human Computer Interaction, which is built around the ability to help users take advantage of an interface to perform a task. In this subfield aesthetic principles such as the interface's usability, learnability, and invisibility are essential. On the musical side of things, composition of new works occurs quite often through the objective framework of modern music theory, in which some chords and progressions follow logically from others.

From here it stands to reason that music and computer science have a lot more in common than they would appear to at first glance, and that neither is exclusively subjective nor objective. As such, this paper will be centered on a case study designed to probe a possible objective method for analyzing music that is already proven itself in the analysis computational systems: information theory. More specifically, this study has been conducted through the collection of data on information entropy in the genre of shape note music. The genre of shape note music has been chosen for its relative simplicity compared to other genres such as the classical symphony, which will render it significantly easier to analyze objectively. Information entropy as proposed by Claude Shannon, makes an excellent method for objective analysis

because it deals directly with probability and uncertainty, which are always present in a field such as music that depends so heavily on human subjectivities.

Over the course of this study, data on pitch and rhythmic entropies are collected for each of the three or four voice parts in a sample of 20 shape note tunes from *The Sacred Harp*. These entropy values are then put through rudimentary statistical analysis to determine if any pattern or correlation emerges among the entropy values within the shape note tunes. If any do, this would imply that at east some elements of shape note music can be modeled and expressed using information entropy. Such a finding would create ground for further research into the possibility of modeling musical systems using information entropy.

2. Review of Literature

2.1 Computation and Music

In order to analyze western music and digital computation effectively, it is necessary to have some concrete way to conceptualize and compare them. For digital computation, professors Jan van den Ende and René Kemp propose modeling it as a technological regime that emerged through a co-evolutionary process. A regime, according to Ende and Kemp, " is defined as the grammar or rule-set embedded in the coherent complex of a technology (or mode of manufacturing) which structures the search activities of engineers and the policies and actions of other technology actors (including public authorities)" (1999). In other words, a regime is the set of rules and common practices that govern how a technology functions, and how it is used.

As Ende and Kemp assert, the digital computation regime can thought of as emergent from other, preexisting computing regimes through a co-evolutionary process. Hence, digital computing is not a next step in technological evolution that renders all of the preexisting regimes obsolete. Instead, the digital computing regime developed alongside its predecessors to fill similar demands for computational power, efficiency, and accuracy. The regimes from which digital computing emerged, each named for the technology they represent, were the manual/mathematical instrument, scale model, analogue computing, small office technology, and punch card regimes (Ende and Kemp, 1999).

Digital computing and its predecessor regimes have their roots in formal logic. As Paul Lawrence of the Royal Society tells us, “A *logical formalism* is a precisely defined symbolic language that includes logical primitives such as ‘and’ (∧), ‘or’ (∨), ‘not’ (¬), ‘implies’ (→), ‘for all’ (∀) and ‘there exists’ (∃)” (2018). Formal logic is highly useful in verifying that a given computing system will work the way it was designed to. The branch of computation tailored to digital computing specifically is known as computational logic, and is the principle by which computer hardware is designed. The structure of computer hardware in turn affects the design of the software that runs on it.

Like digital computing, western music of today is also based heavily on theoretical practices. This theoretical system is known as tonality, or the tonal system of music, and has been in use since the late Baroque Era. In more formal terms, tonality can be defined as “The system, common since the late seventeenth century, by which a piece of music is organized around a TONIC NOTE, CHORD, and KEY, to which all their notes and keys are subordinate” (Burkholder et al., A20). As with formal logic, the tonal system defines best practices for the relationships among various elements in a work of music. Moving from the dominant (V) chord to the tonic (I) chord to complete a phrase can be considered a perfect authentic cadence, while moving from the mediant (III) to the supertonic (ii) to create a cadence (proper phrase ending) is simply incorrect practice. Hence, it is clear that both digital computing and tonal music are based on theoretical practices, which can be analyzed and compared.

In addition to being based in theory, western music also has a concept strikingly similar to that of a technological regime. In much the same way that technological regimes are nearly as old as technology itself, musical genres have been recognized for almost as long as musical practices have been written down. As Burkholder et al. point out in their textbook *A History of Western Music*, "Word lists from ca. 2500 BCE on include terms for instruments, tuning procedures, performers, performing techniques, and genres, or types of musical composition" (2019). Furthermore, musical genres tend to emerge from preexisting ones through a similarly coevolutionary process. The instrumental concerto, for instance, evolved out of a combination of elements from the vocal concerto style and the sonata, both of which were well-established musical genres at the time. This came about because instrumental concertos were based on the concertato medium but filled many of the same roles as sonatas (Burkholder et al., 2019). In this way, musical genres can be compared to technological regimes since both define a set of rules and common practices, and both arise from similar coevolutionary processes.

At this point, we have established that the most popular western music since the late Baroque Era can be compared to digital computation on the grounds that each has an underlying objective theory behind it. For western music since the late Baroque, this theory is tonality, while for digital computing it is computational logic. We have also established that musical genres, including those classified under the broader umbrella of western music, can be viewed in much the same way as technological regimes, such as digital computing. It follows from these premises that current genres in western music can be compared to the digital computing regime through comparison of the objective rules and common practices found in each. This conclusion naturally begs the question of "How can a comparison of genres/regimes with completely different theoretical bases actually be made?" One possible solution is to find a common thread between them and use it as a point for comparison. For the purposes of this research, that common thread will be information theory, and the aim will be to investigate the question of whether or not musical systems can be modeled objectively using the concept of information entropy.

2.2 The Aesthetic Qualities of Music

Before delving deep into the realm of information theory, it is important to first define the aesthetic qualities of western music, as they will be discussed in this research, and define their relationship to the previously established concept of the musical genre. The aesthetic qualities of music are those that contribute to its overall appeal to the listening audience. While there are many different technical terms to describe these qualities, it is helpful to group them into several broad categories. Jane Cassidy and Donald Spear of Louisiana State University offer one particularly helpful categorization in their study on the effects of differential training on the use of musical terminology in a concert review. According to Cassidy and Speer, most musical terminology falls under the following six categories: instrumentation, tempo, melody, harmony/texture, form, and dynamics. These are not the only possible categorizations for musical terminology, but they do cover almost all of its practical use cases.

Another important thing to note about their study is its finding. In comparing the use of musical terminology in students who were not music majors both before and after some terminology training, Cassidy and Speer discovered that training students with the vocabulary alone made far less of a difference in their usage of musical terminology than when vocabulary training was paired with a listening component (1990). This means learning musical terminology through training that included an auditory component had a much greater impact on a student's likelihood to use terminology in certain categories. In particular, this resulted in a sharp decrease in the likelihood of students to use instrumentation related terminology in the group trained with an auditory component, and a considerable increase in likelihood to use terminology related to tempo, melody, and dynamics that was unmatched in the other group (Cassidy and Speer, 1990). Such a finding indicates that the combination of vocabulary and audio training had a diversifying affect the music terminology used by its trainees that was unmatched by vocabulary training alone.

This may imply that without the ability to describe music's aesthetic qualities through firsthand experience, audiences end up hearing a lot more in music than they can consciously understand and articulate. This may also lend credence to the idea that the aesthetic qualities in music govern our understanding of it, given that those who did learn from firsthand experience were quickly able to use newly learned terminology to describe the same concert experience. Hence, the ability to tie together the advanced computer programs of the modern digital computing regime and the musical aesthetic produced within various musical genres in the context of one unifying theory stands to shed a lot of light on the relationship between digital computation and western music.

2.3 Shape Note Music

The Shape Note system for learning, reading, and singing music is a genre of American sacred music that originated in the New England region and became extremely popular in the South as well. First codified in 1801 with the publication of William Little's songbook, *The Easy Instructor*, shape note music was exclusively vocal. This is largely because it was composed for use in the church services of Protestant denominations, such as the Baptist Church, that did not allow the use of instruments in their services. As a result, shape note tunes are typically written for an entire congregation that is divided into three or four separate groups, each with their own part to sing. These three or four parts are typically referred to as the Soprano/Treble, Tenor, and Bass (STB) in the three-part format, or Soprano/Treble, Alto, Tenor, Bass (SATB) in the four-part format.

So far as pitch is concerned, the Soprano/Treble singers will generally sing the highest notes, the Bass section will sing the lowest notes, and the Tenors will be in between, carrying the main melody. For SATB format, the altos generally sing higher than the tenors who carry the melody, but lower than the Sopranos, but in both STB and SATB formats, voice parts will sometimes overlap in terms of pitch. This means that the relationship among these voice parts is not absolute throughout the entire tune, and there may be moments when the Alto sings higher than the Soprano, for instance, before falling back below it. Crossing voices in this way was not uncommon in western music at the time, so it is no surprise that shape note music also does this.

Theoretically speaking, shape note music uses a system of tones classified into two main modalities: the major modality and the minor modality. In terms of relative pitch, these correspond precisely to the sequences of half steps and whole steps that defined the major and minor scales respectively, which were used in most western music at the time of its origin. Pedagogically, however, these tones are taught using somewhat different notational and phonetic systems from those of other western vocal music at the time. Phonetically, both systems use some form of solmization, which is a system for associating notes with spoken syllables. The set of syllables used in a particular solmization are known as its solfege. For most western vocal music, the seven notes within an octave are phonetically taught using the seven syllables solfege of *do, re, mi, fa, sol, la,* and *ti,* with the new octave starting back on the next *do*.

Shape note music, on the other hand, is taught using a four syllable solfege known as fasola solmization to model the octave, which runs, *fa, sol, la, fa, sol, la, mi,* before returning to the original *fa* at the octave and repeating the pattern. This alternate solfege is extremely important in understanding the notational deviation that sets shape note music apart from other forms of vocal music. Most western music, both vocal and instrumental, is written on a five line staff with notes that have uniformly shaped, rounded heads positioned either on a line, or in the space between lines. Pitch is absolute and indicated by a combination of a clef symbol (usually a c, g, or f clef), key signature, and the position of the note head.

In shape note music by contrast, pitch is relative, and set by whoever is leading the congregation in song. In place of rounded note heads, the note heads in shape note music use a set of four distinct shapes, with one corresponding to each unique syllable in the shape note solfege. These unique shapes render it unnecessary to learn the musical staff in order to read shape note music. Rather than relying on a mental mapping of note based on the combination of clef, key signature, and position of the note head on the staff to read music, shape note singers can read music simply by looking at the shape of the note, and whether its staff position is higher or lower relative to the previous note. This makes shape note music intellectually accessible to a demographic of singers with no formal musical training, who merely learned the shape note solmization system at some point in their lives.

Additionally, shape note music is typically written out on five line staff with a clef and key signature. This practice makes it instantly legible to anyone with formal musical training as well. Take the tune *Mount Pleasant* from **Figure 1** below for example.

Figure 1 - *Mount Pleasant* from the 1860 version of *The Sacred Harp* (White and King) published by Collins Printing House.

The soprano and tenor parts are written with a treble clef, while the alto part uses the alto clef and the bass uses bass clef. All share the same key signature with exactly one flat, indicative of the key of F Major in this case, and are written out on a 5 line staff. Although pitch is relative in shape note music, and a starting pitch is typically given by whoever is leading the congregation in song, these notational markings provide the classically trained musician with a point of reference from which they can read and interpret the tune without needing to know shape note solfege. At the same time, each note head has a characteristic shape corresponding to its solfege syllable, with the sopranos starting on the oval shaped *sol*, the altos on the square shaped *la*, and the tenors and bass singers on the triangle shaped *fa*.

In practice, shape note tunes such as *Mount Pleasant* are typically sung by a congregation seated in a square shaped formation. When written in SATB format, this means each side of the square will consist of singers from each of the four voice parts. All singers will sit facing the center of the square, and one another as a result, where a conductor will stand to lead them in song. It is typically done this way because Shape Note singing "is a participatory activity, and not a performance, the singers arrange their seating so as to focus the sound inward, toward the center of the group, instead of projecting it outward toward an audience" (Steel, 2016). The conductor will give the starting pitch relative to which singers will tune themselves before immediately entering into song. It is also the conductor's responsibility to beat time, which means dictating the tempo through a simple repetitive arm motion. Singers facing the conductor will also beat time with this arm motion. This is done both in solidarity, and for the benefit of those seated behind the conductor's back at any given moment, since facing the conductor also means facing them.

Hymns are almost always read out of tune books, which contain large collections of individual shape note tunes. Not surprisingly, many of the tunes in these books are derived from folk songs that have been adapted to fit the major and minor modes of shape note music. However, these folk tunes often predated the tonal system of music, on which shape note notation and solmization are based. Instead, some of these folk tunes had been composed according to gapped scales, and others according to the modal system of harmony, which was tonality's predecessor. Such folk tunes had to be adapted somehow in order to fit the tonal system shape note music was based on. According to Schwob School of Music Director Daniel Taddie, "[g]apped scales present no difficulty, since they simply omit notes from the diatonic scale" (1996). In other words, they feature some, but not all of the pitches on a tonal scale.

Modal tunes on the other hand, presented a much greater challenge. Like tonal scales, modal ones do have a root note and repeat note names at the interval of an octave (e.g. A3 is one octave below A4). However, modal music does not feature the concept sharp or flat notes that is essential to the tonal system, with the notable exception of B^b, which was used in some modes to avoid writing a dissonant tritone interval into the scale. Hence, there are far fewer modes in the modal system than there are keys in the tonal one. A tonal key such as E^b Major, for instance, has no direct modal equivalent nor parallel, and the Dorian mode is not the same as the D Major or d minor scales, even though all three share D as their root note. That being said, there are a couple of modes that are easy to transpose into tonal keys. The Ionian mode, for example, has the exact same scale as C Major, and having the same intervals between its notes as a Major scale, can be transposed readily into any major key.

Although some modes from the modal system lined up quite well with the tonal major and minor modes of the shape note system, "the key theory in the tune books did not account for modes other than major or minor" (Taddie, 1996). As a result, folk tunes originally written in the Mixolydian, Phrygian, and Dorian modes proved difficult to adapt and transpose. As Taddie tells us, "[t]he clue to accommodating modal scales within the shape-note stylistic conventions lies in statements about the key note. Nearly all tune books state that the key note is to be found in the last note of the bass" (1996). The key note lies at the exact interval of a fifth below the root note for the adapted tune's mode, and serves as the root note for the shape note tune's key. In the Mixolydian and Phrygian modes, the root notes are G and E respectively, following the scales outlined in **Figure 2** below.

Mixolydian scale: **G – A – B – C – D – E – F**

C Major Scale: **C – D – E – F – G – A – B**

Phrygian scale: **E – F – G – A – B – C – D**

A minor Scale: **A – B – C – D – E – F – G**

Figure 2 – Comparison of Mixolydian and Phrygian Modes with C Major and A minor Scales Respectively

For both modes, the key note (green underlined notes in **Figure 2**) is the fourth scale degree, which when counting backwards, turns out to be the fifth below the root note (highlighted yellow in **Figure 2**) of each mode. As can be seen in **Figure 2**, the C Major scale can be expressed as the Mixolydian mode altered so that the key note is now the root note. The same relationship occurs between the A minor scale and the Phrygian mode. In both cases, the original root note of the mode is now the fifth degree of each tonal scale. In tonal theory, this makes it both the dominant scale degree, and a member of the tonic chord, which is the chord built out of the root (also called the tonic) note in the scale. Because of this relationship, it is possible to express the Mixolydian and Phrygian modes in terms of a Major and a minor key respectively. This allows Mixolydian tunes to be transposed into any key within the major mode, and Phrygian ones into any key within the minor mode, without losing the modal melody.

This can be seen in Mixolydian tune *Halleluiah* (**Figure 3**), from page 146 of the 1860 edition of The Sacred Harp that is pictured below. Notice how the tune itself is harmonized in the key of B^b Major, while the Tenor melody sticks more closely to the key of F Major, the dominant of B^b Major. Also note that the Soprano and Bass lines both end on the key note (tonic) of Bb, while the Tenor ends on the modally derived root note of F. This tune has been transposed from the Mixolydian mode and key of C Major to the keys of F and B^b Major respectively.

Figure 3 – *Halleluiah*, a Mixolydian tune from the 1860 version of *The Sacred Harp* (White and King) published by Collins Printing House.

In practice, Taddie tells us that most shape note tune books will transpose these modal folk melodies, most often to match a "sharp key," and use them in the tenor voice. That tenor melody can then be harmonized in the other three voices key indicated by the transposed key note (1996). This will result in the tenor voice ending on the root note of the mode, as is typical in modal harmony. That root note for the mode, will also turn out to be a part of the tonic chord in the tonal key, thus fitting in harmonically with the other voices in the tune, which end on either the key note (root of the chord) or the third of the key note chord. The Dorian mode, on the other hand, cannot be harmonized in this way. Instead they are typically "written in Aeolian mode [which lines up with A minor] but performed in Dorian mode by introducing an unwritten raised sixth degree, without changing the solmization syllable" (Taddie, 1996). In other words, Dorian tunes are simply performed slightly differently from their notation to accommodate for the differences between shape note notation and modal theory.

2.4 The Origin and History of Shape Note Music

As mentioned above, shape note music originated in the New England region of the United States, having its roots in New England psalmody. New England Psalmody is the practice of singing metrical versions of Old Testament Psalms that were brought to the New World by the English Separatists, also known as the Pilgrims (Guthrie, 2014). Because of this, New England Psalmody has its roots in English folk music and music used in 17th Century protestant church services. The first notable tune book to feature New England Psalmody was the *Bay Psalm Book*, which was published in 1640.

Singing by rote dominated New England congregational practices during the 17th and early 18th Centuries due to widespread illiteracy in the colonies, and a comparable lack of education on how to read music. However, this resulted in what is best described as "poor congregational singing led by preachers and ministers" (Guthrie, 2014), who likely had little formal musical training themselves. This fact led many 18th Century New England preachers and ministers to advocate a return to singing by note, and it suddenly became imperative to find a pedagogical system to meet this demand. As a result, many instructional books were published throughout the Century in the effort to find an efficient way to teach music.

One of the earliest and most noteworthy of these instruction books was Rev. John Tufts's 1721 publication, *An Introduction to the Singing of Psalm Tunes in a Plain and Easy Method*. In this instruction book, "Tufts placed letters on the staff to represent the four solmization syllables of fa-sol-la-mi" (Guthrie, 2014). Such a system of lettering each note according to solfege served as a precursor to the Shape Note system, which would end up using the shape of each note head to represent its solmization syllables rather than the first letter of the syllable. Shape Note singing itself would first emerge in 1801, through Little and Smith's tune book *The Easy Instructor*.

Due to their popularity, *The Easy Instructor* and other shape note tune books that followed shortly afterwards led to the development of singing schools. Singing schools were almost exactly as their name would imply. They were institutions devoted to vocal pedagogy at a time before music was taught in public schools, and shape note tune books were often their textbooks. In the north, however, the enthusiasm for shape note music was somewhat short lived. This was due largely to the influence of musical reformers such as Lowell Mason, who pushed for a "learned approach to music as used by the European music masters" (Guthrie, 2014). Hymns written by these reformers were straightforward with their harmony and written mostly as homophonic tunes, as opposed the modal harmony and polyphonic writing of preexisting shape note tunes. Singing schools also began to disappear as the 19th Century progressed, being replaced by the inclusion of music in public education.

That being said, declining popularity in New England did not stop shape note music, and the singing schools that came along with it, from spreading to the South and West. It was here in the South and along the Western Frontier that shape note music became part of the social lives of its singers both inside and outside of church walls. Some of the most famous tune books to emerge from the South included Ananias Davisson's *Kentucky Harmony* (1815), Carrell and Clayton's *Virginia Harmony* (1831), William Walker's *Southern Harmony* (1835), and White and King's *The Sacred Harp* (1844). As the musical reform movement championed by Lowell Mason reached the regions, shape note music did decline in popularity in the South and West as well. However, there are still isolated regions of the South where shape note music never died out, remaining an integral part of social life in the local communities even in the 21st Century.

2.5 Form, Purpose, and Popularity

As much as shape note tunes have in common, there are also a couple of key differences between tunes within the Shape Note genre that set some apart from others of their kind. Two such differences are in the form and purpose of the tunes. A tune's form is the overall structure that it follows, whereas its purpose refers to the reason for which a tune was composed and sung. So far as form is concerned, shape note tunes fall overwhelmingly into the following categories: psalm tunes (in the style of New England psalmody), hymns, fuguing tunes, odes, and anthems. Psalms in shape note music are tunes created from metrical arrangements of Old Testament texts. Hymns on the other hand, are songs written for performance in church services with text that praises God, but is not drawn directly from the Bible. Examples of a shape note psalm and a shape note hymn can be seen in *Huntington* (**Figure 4)** and *Missionary Hymn* **(Figure 5)** respectively.

Figure 4 - *Mount Huntington,* a Metric Psalm from the 1860 version of *The Sacred Harp* (White and King) published by Collins Printing House.

Figure 5 – *Missionary Hymn,* a Hymn from the 1860 version of *The Sacred Harp* (White and King) published by Collins Printing House.

Fugiung Tunes, as seen in *Stratfield* (**Figure 6)**, feature a section where a theme or musical idea is repeated across multiple voices, much like the repeated thematic material seen in the European Fugue. In the case of *Stratfield*, the theme is a simple sequence of 5 quarter notes that first appears in the Bass line on the pickup to measure 8, rises to the Tenor at the pickup to measure 9, skips to the Soprano at the pickup to measure 10, and repeats itself for the final time in the Alto at the pickup to measure 11. This repetition of a small theme is what characterizes *Stratfield* as a fuguing tune.

Figure 6 – *Stratfield*, a Fuguing Tune from the 1860 version of *The Sacred Harp* (White and King) published by Collins Printing House.

Odes are the result of setting poetry to a vocal melody, while Anthems are compositions with religious or political lyrics. Examples of an Ode and an Anthem can be seen in *Ode of Life's Journey* (**Figure 7)** and in *Christmas Anthem* (**Figure 8)** respectively. As implied by their titles, *Ode to Life's Journey* draws its lyrics and melody from the setting of a poetic text, while *Christmas Anthem* is religious in nature.

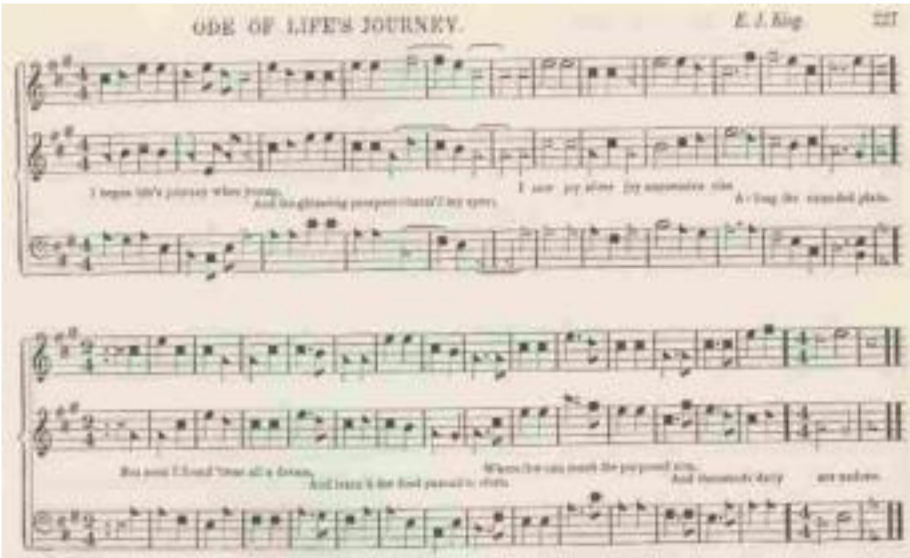


Figure 7 – *Ode to Life's Journey*, an Ode from the 1860 version of *The Sacred Harp* (White and King) published by Collins Printing House.

Figure 8 – *Christmas Anthem*, an Anthem from the 1860 version of *The Sacred Harp* (White and King) published by Collins Printing House.

In terms of purpose, all shape note tunes are meant to be sung by a large group as a communal activity. However, there are some key differences among shape note tunes in that some are written primarily for singing at church services, while others are written for singing schools and societies. These two separate purposes are the ones that will be addressed in the **Methods**, **Results**, and **Discussion** sections of this study.

In addition to differing in form and purpose, not all shape note tunes are equally popular. Some, in fact, are sung much more frequently than others. Unlike form and purpose, shape note tunes are not composed based on their popularity. Popularity is by contrast an attribute that tunes take on after they have already been composed and introduced to the general public. Because of this, popularity is an excellent metric for exploring the affect of a tune's entropy on another variable via correlational data. By contrast, data collected on form and purpose will be used to examine the affect of composer's choices and intentions on the resulting entropy in the tune.

2.6 The Sacred Harp

First published in 1844, The Sacred Harp is the most well-known and widely used shape note tune book today. It has undergone numerous revisions since the first version in 1844, including the Collins Printing Press edition of 1860 used in this study, which keeps all of the tunes from the original, but adds appendices that include additional shape note tunes. Structurally speaking, the tune book spans pages divided into three parts, Part I, Part II, and Part III followed two appendices, Appendix I and Appendix II. Part I and Part II are delineated from one another in terms of the purpose for which tunes in each one are written to be sung. Part I is titled *Consisting of Pieces Used By Worshipping Assemblies*, and contains tunes that are meant to be sung during church services or other religious ceremonies. Part II on the other hand, is titled

Consisting Principally Of Pieces Used In Singing Schools And Societies, and contains tunes written primarily for pedagogical use and/or recreational singing outside of religious services.

While Part I and Part II both contain mostly hymns and psalms with the occasional fuguing tune, Part III differs from its predecessors in form. Its title, *Consisting of Odes and Anthems*, is fairly self-explanatory as to which forms it primarily features. Labeled simply as *Appendix to the Sacred Harp*, the first of this edition's two appendices is devoted to tunes that were either popular or considered standards of the genre, but "not comprised in the body of the work" (White and King). In other words, it contains tunes that were popular or otherwise famous at the time of its publication. The second appendix, labeled as *New Appendix*, contains tunes that were new compositions at the time of its publication. It is worth noting that the inclusion of these two appendices is the primary difference between this edition of *The Sacred Harp* and the original 1844 edition.

2.7 Information Theory

In order to model the aesthetic qualities of music and computer programs as information systems, it is necessary to have a basic concept of what an information system is, and how to model one in the first place. Information Theory is an excellent framework for modeling information systems of any kind and will be used for the sake of this project. Modern information theory is an abstract concept that is concerned chiefly with point-to-point communication. The foundations of information theory were first articulated by Claude Shannon in 'A Mathematical Theory of Communcation' (Shannon, 1948).

As University of Maine Professor George Markowsky explains, the are five major components to Shannon's model of information are a message's source, encoder, channel, decoder, and receiver. At the beginning of communication, the source is the entity that first creates the message, and the encoder is "the object that connects the message to the actual physical signals that are being sent" (Markowsky, 1998). If the message were a spoken English sentence for example, the source would be the mind of the person speaking the sentence, while the encoder would be the speaker's mouth as they formed the words of the sentence. The sentence would then be transmitted through the air, which would be its channel, in the encoded form of a sound. More generally, the channel for a message is any medium that carries it between sender and receiver. Channels can be interrupted by noise, which can be defined as "anything that interferes with the transmission of a signal" (Markowsky, 1998).

Upon crossing the channel, a message in transit will arrive at the decoder, which essentially performs the reverse of the encoding process by translating the message sent by the source into a form that the receiver can understand. In the case of the English sentence, the decoder would be the auditory system of the person being spoken to, which translates the sound waves arriving at the ears into a message that can be understood by the brain of the person being spoken to. Not surprisingly, the receiver in this case is the mind of the person being spoken to, which gets the message that is the sentence.

In addition to the abstract model describing the process point-to-point communication, Shannon also proposed several key ideas about the information contained in messages travelling from source to receiver. One of these is the concept that information is uncertainty, and its value stems from uncertainty. As Professor Yeun of the Chinese University of Hong Kong explains it, "if a piece of information we are interested in is deterministic, then it has no value at all because it is already known with no uncertainty" (2008). Take a hypothetical violinist, for example, who delivers a performance that consists only of playing the note A4 repeatedly in quarter notes for several minutes with no variation whatsoever in rhythm, tempo, dynamics, nor articulation. Such a performance would lose the attention of most audiences very quickly, as they became increasingly certain that quarter notes of A4 were all that was really going to happen. Furthermore, anyone who had bought tickets to that performance would probably want their money back at the end, since they received nothing of from that droning, monotonous lack of uncertainty. Because uncertainty is this critical to the value of information, it follows that information can be described mathematically through a combination of random variables and probabilities.

Another core concept about information is the idea that information is digital. According to Yeun, this means that information delivered in point to point communication can be encoded in binary "into a stream of 0s and 1s called bits, and the remaining task is to deliver these bits to the receiver correctly with no reference to their actual meaning," (2008) which is decoded upon receipt. This is turns out to be the fundamental concept behind communication in computer networks. The concepts of entropy and capacity as fundamental measures of information also arise from Shannon's work. Entropy, or the natural degree of uncertainty within a given system, is described by his source coding theorem and sets the limit for the most efficient rate of error free communication achievable in a given information system. Capacity emerges from his channel coding theorem, and denotes the fastest rate of communication achievable when noise in the communication channel is factored in.

2.8 Historic Precedent for Use of Information Entropy Musical Analysis

Entropy in general has multiple technical definitions, depending on the field that the concept is being applied in. For the purposes of this study, Claude Shannon's definition of information entropy will be used in the analysis of shape note music. That being said, this will not be the first time information entropy has ever been used in musical analysis. In fact, researchers have been drawn to the use of entropy among other information theory concepts since shortly after Shannon's 1948 papers were published. As Marcus Pearce of the University of London puts it, "researchers used information-theoretic concepts [including entropy] and methods throughout the 1950s and 60s both to analyse music (Cohen, 1962; Meyer, 1957) and to generate new compositions" (2007). That trend has not ceased entirely between then and now, as information entropy remains an important concept across many disciplines today. In music, however, entropy-based analysis has largely fallen out of the mainstream since the 1960's, meaning much of the literature on attempts to apply it to musical works has since become dated.

3. Methods

3.1 Defining Pitch and Rhythmic Entropies

As mentioned above, entropy itself is a measure of the degree of uncertainty in a system. Shannon's entropy in particular measures entropy in terms of the bit, which can be thought of as the answer to a binary, yes or no/on or off type question that helps identify the value of a given element. For pitch and rhythmic entropies, this will mean measuring the average uncertainty, in bits, of the pitches and rhythms within shape note music. In order to calculate these entropy values, both pitch and rhythm within each shape note tune will be analyzed according to Shannon's formula for information entropy:

$$H = -\Sigma\ p(x)\log_2(p(x))$$

Where p(x) represents the probability of value x occurring and the summation (Σ) is done over the set of every possible element in the system. For pitch entropy, this will be every possible pitch value in the shape note tune, and for rhythm, it will be every unique rhythmic pattern (e.g. quarter note, eighth note, etc.).

3.2 Determining Sets of Possibilities

Pitch and rhythmic entropies are determined by a formula that factors in the probabilities of each individual rhythm or pitch occurring in the system being analyzed, which is currently defined loosely as the shape note tune under analysis. To make this possible, it stands to reason that there must be a concrete way to define the set possible values for each of pitch and rhythm. For the purposes of this study, one such method will be used for the set of possible pitch values, and another will be used to determine the set of rhythmic values.

In the case of pitch, possible values are distinguished using the International Standards Organization (ISO) system of pitch designations, in which the pitch produced by playing middle C on the piano is notated C4. As Open Music Theory's forum points out, one important detail to note about the ISO system is that "[t]he tricky bit about this system is that the octave starts on C and ends on B." In other words, the pitch designation for one half step below C4 is actually B3, and the note one half step above B4 is C5. This detail is important in understanding the pitch designations used to identify unique pitch values in the raw data (Appendix 1). For the purposes of this study, rests will also be counted as unique pitch value with a pitch of silent (0Hz), but a computable probability of actually occurring. This distinction owes to the fact that rests and the silence they entail are a structural part of music, regardless of genre.

In the case of rhythm on the other hand, possible values are determined by pitch duration and notated in the raw data (Appendix 1) according to rhythmic markings. Possible markings include, but are not limited to the whole note, half note, quarter note, eighth note, and sixteenth note, as well as dotted and tied notes. In the case of a tie, which is one pitch value being marked at multiple rhythmic values that are all grouped together under an arch (the usual indication for a slur), the tie itself (and hence all markings under it) will be counted as one rhythmic value. This is because tied notes are actually held for the total duration of all rhythmic markings under the tie. Consecutive rests will be counted as if though are a tie on the rest note value. Slurred notes, however, which are marked as notes with distinct pitch values and rhythmic notations under an arch are still all considered rhythmically unique, and have separate values. Below key for how understanding how rhythm values were notated in the raw data (Appendix 1):

<u>Key for Rhythmic Notation</u>

W = Whole Note

H = Half Note

Q = Quarter Note

E = Eighth Note

S = Sixteenth note

D = dotted (ex. DH = Dotted Half Note)

T = tied to (ex. W**T**Q = Whole Note Tied to Quarter Note (5 beats))

*Ties are counted as a single note

*Consecutive rests are considered as one note value equal to the duration of the rest (This could be counted as a tie)

*Repeats for multiple verses are assumed, but entropy is only altered by changes in the number of distinct note values in the piece and their relative frequencies. This means that repeats will have no impact on the result of an entropy calculation since they do not alter the probability of one pitch or rhythmic value occurring on an arbitrarily chosen note, and are factored out of the frequency data for simplicity and ease of calculation.

3.3 Distinguishing Unique Note Values

With the systems all possible rhythmic and pitch values used in data collection established, there is one more essential question left to answer: In terms of the actual markings on the page, what determines a unique value, as opposed to a set of multiple markings that can be grouped together under one value, as discussed earlier in the case of ties? In this study, the answer will be that distinct values are determined on a note-by-note basis. The note for these purposes will be defined as a marking or group of markings that has a distinct pitch or rhythm value from its immediate neighbors and is articulated separately from all other notes. Markings under a tie, for instance, do not meet this definition on their own since they are not articulated separately, and must be viewed as a group. The same is true for consecutive rests, since silence does not rearticulate itself audibly. This means that multiple rest markings, such as a half note rest followed by a quarter note rest, do not constitute two separate notes for analysis.

3.4 Defining and Analyzing the System

With sets of unique note values defined, it is now imperative to define the system over which our entropy values are being calculated in more specific terms than simply "the shape note tune as a whole." Instead, using our definition of a unique note value, the system, or shape note tune can be defined more narrowly as a set of (usually 3 or 4) sequences of distinct notes that occur simultaneously. Each of these sequences represents one of the voice parts that comprise the tune as a whole, that are usually notated in an SATB (Soprano, Alto, Tenor, Bass) or STB (Soprano, Tenor, Bass) format. For ease of analysis, the shape note tune can be further broken into subsequences represented by each voice, with each subsystem being analyzed according to the number of notes bearing each pitch value that occurs, and the number bearing each rhythm value.

Probabilities for each pitch value can then be determined by dividing the number of occurrences of each pitch value by the total length of the sequence. The same process can be repeated with rhythmic values to determine their relative probabilities. If the pitch A4, for example, occurs on 11 distinct notes in a Soprano voice that contains 55 notes in total, the probability of A4, p(A4) for that voice is 11/55, or 20%. This probability will enter the pitch entropy equation as the term

$$\mathbf{p(A4)\log_2(p(A4)) = (.2)\log_2(.2)}$$

It will be a term in the summation (Σ) written out to include one term for every pitch that occurs in the soprano voice. For the purposes of this study, that process is completed twice for each voice in the 20 shape note tunes analyzed, producing values for the pitch and rhythmic entropy of each one. These values are then subjected to statistical analysis to paint a picture of entropy in shape note music at a low level of abstraction: that of individual notes that make up each voice in the musical work.

3.5 Rationale

Given the low level of abstraction this study chooses to focus on, it is only natural to ask why. For what reason is this level of abstraction chosen in the first place, and why does it not also analyze others? The question also remains as to why pitch and rhythm are chosen as the elements for analysis in the first place. Why not others such as tempo or harmonic/chord structure? The answers to these questions boil down to a combination of the generalizability of the study's methods, and the goals and the limitations of the study itself. Limitations are factor because this study is not designed to paint a comprehensive picture of the entire relationship between entropy and music at all levels of abstraction. This would simply be beyond the scope of the project. This fact among other limitations will be discussed in greater depth in the subsection of the study's **Discussion** section titled **Limitations**.

Instead, the design of this study is geared towards the generalizability of its methods, because it is intended mainly to probe into the potential for a relationship between entropy and shape note music, and to serve as a launching point for further research should it produce significant results. This means that this study is intended not only to be replicated, as virtually all scientific studies are in principle, it is also intended to be adapted into similar forms that probe into a potential broader relationship between entropy and music. To this effect, the note-by-note level of abstraction lends itself to this generalizability in ways that significantly higher levels of abstraction might not.

Take chord structure for example. Chords and chord progressions can be considered a higher level of musical abstraction than the notes and note sequences that they are comprised of. At this level of abstraction however, different musical genres have differing systems for defining what chords are even possible in the first place. The chord structures modal genres from the Middle Ages such as Gregorian Chant, for instance, do not support the concept of a V^7/V chord in G Major the way modern tonal music would. As a result, methods for calculating the harmonic entropy of shape note music cannot be reapplied as broadly across varying musical genres as those doing the same with note sequences. For this reason, the methods analysis at the chord and chord progression level of abstraction are less generalizable to other experimental contexts than the note and note sequence level ones in this study.

That being said, abstraction level is not the only factor limiting the generalizability of methods that focus measuring entropy in one particular aesthetic quality of music. Recall, for example, that Cassidy and Speer mention six broad classifications for the aesthetic qualities of music: instrumentation, tempo, melody, harmony/texture, form, and dynamics. Of these, form was ruled out for being at too high a level of abstraction. Tempo and dynamics on the other hand, were ruled out because they are not notated in shape note music from *The Sacred Harp*, and thus cannot be measured and compared to anything by reading the score. Instrumentation was also ruled out since it is nearly a constant across shape note tunes, which are generally written for ether 3 or 4 vocal parts sung by a religious congregation. This leaves melody and harmony, which can both be analyzed in terms of the note sequences in the melody and harmony voices. Although harmony can also be analyzed at the chord structure level, and both can hypothetically be analyzed in terms of note patterns such as musical sequences and phrases, such methods were ruled out in favor of time and resource constraints.

3.6 The Data Collection Process

For the purposes of this study, 20 shape note tunes were drawn pseudorandomly from the original 1860 edition of *The Sacred Harp* (available on IMSLP) and analyzed for their pitch and rhythmic entropies. 10 pieces were selected from each of Part I and Part II of *The Sacred Harp* using code (Appendix 2) that applied Java's pseudorandom number generation algorithm to the structure of the 1860 edition. By running the program, a researcher would be able to specify a section in *The Sacred Harp* (e.g. Part I or Part II), and receive randomly generated values for a page number and position on the page (either 1 or 2). The page value would instruct the researcher to select a tune from that page in The Sacred Harp, and the position would indicate which tune to choose if two pieces shared a page. These are the values that appear next to the titles of each shape note tune in the raw data from Appendix 1. Position is ignored if there is only one tune on a page, and pos. 2 is sometimes struck through to indicate that the value was generated but not used. This pseudorandom method for tune selection helped to eliminate any potential bias in the selection process.

Once a shape note tune was selected, the following data was collected on each of its three or four voices:

1. The length of the voice part in notes
2. The set of all distinct pitches in the part
3. The set of all rhythmic values (quarter note, eighth note, etc.)
4. The frequency with which each pitch and rhythmic value occurs
5. The popularity ranking for that tune when provided by the Sacred Harp Musical Heritage Association database – stored in the raw data (Appendix 1) as number behind the '*' characters on title lines for most tunes

The number of notes that have each pitch or rhythmic value determines its frequency. In terms of notation, pitch values are indicated using the International Standards Organization (ISO) system of pitch designations, in which middle C is notated C4. Chords within a single voice partare indicated using the "**&**" to connect two or more pitch designations (e.g. C4**&**G4) and are counted as unique pitch values. In practice, these are usually sung by dividing the group of singers in on a voice part into multiple subgroups that each sing one of the tones in the chord, as in the musical instruction divisi. Tenor parts in the 1860 addition are written in treble clef, and designated accordingly in the data, but are often sung an octave below the written value. This difference does not affect the entropy of the tenor voice since all notes would shift by exactly the same interval from the written value when sung (1 octave). For the purposes of this data set, rhythmic values are marked according to the key from the **Determining Sets of Possibilities** section above. This key will also be included with the raw data in Appendix 1. Additionally, both the set of pitches and the set of rhythms within each voice are displayed here in paired with their frequencies as (pitch, occurrence) or (rhythm, occurrence) pairs.

Falling back on an earlier example, if the pitch A4 occurred 11 times in the soprano voice of a shape note tune 55 notes in length, this information would be stored as the value (A4, 11). Set of pitch occurrence pairs for the tune's soprano voice in the raw data might then look like the following:

(Pitch, occurrence) pairs: {(A4, 11), (G4, 7), (E4, 5), (D4, 3), ($F^{\#}$4, 2), (B4, 15), (C4, 10), (D4, 1), (E4, 1)}

Note that all occurrence values from these pairs total to the voice's length of 55 notes. This should always be the case since by the definition of a distinct note used in this study, every note contributes one occurrence of a pitch value and one of a rhythmic value. This fact was used to check the data for accuracy. From this raw data, pitch and rhythmic entropy values are calculated by applying Shannon's formula for information entropy to this data. Once entropy values have calculated for each voice in the selected tunes these entropy values are organized together and subjected to statistical analysis as discussed in the **Results** section.

During analysis, pitch and rhythmic entropies will serve as the dependent variables, with genre, source, and aesthetic qualities not notated in *The Sacred Harp* including tempo and dynamics will be treated as constants. The purpose of the tunes will be used as an independent variable, since it varies only slightly within the shape note genre, and tunes with slightly different purposes are stored in their own separate parts of the book, with Part I being labeled as *Consisting of Pieces Used By Worshipping Assemblies* and Part II as *Consisting Principally of Pieces Used in Singing Schools and Societies*. Tunes form both Part I and Part II will be analyzed by applying several measures of central tendency to the pitch and rhythmic entropy values collected from their voice parts. These are the mean, median, maximum/minimum values, first and third quartiles of data for each voice part. Of these, all but the mean will be visualized via box and whisker plots to analyze the spread of the data. The significance of the mean values of each data set will be discussed in the subsections dealing with the purpose specific tables and box plots. Potential correlations between popularity and entropy, or form and entropy will also be discussed.

4. Results

4.1 Overview

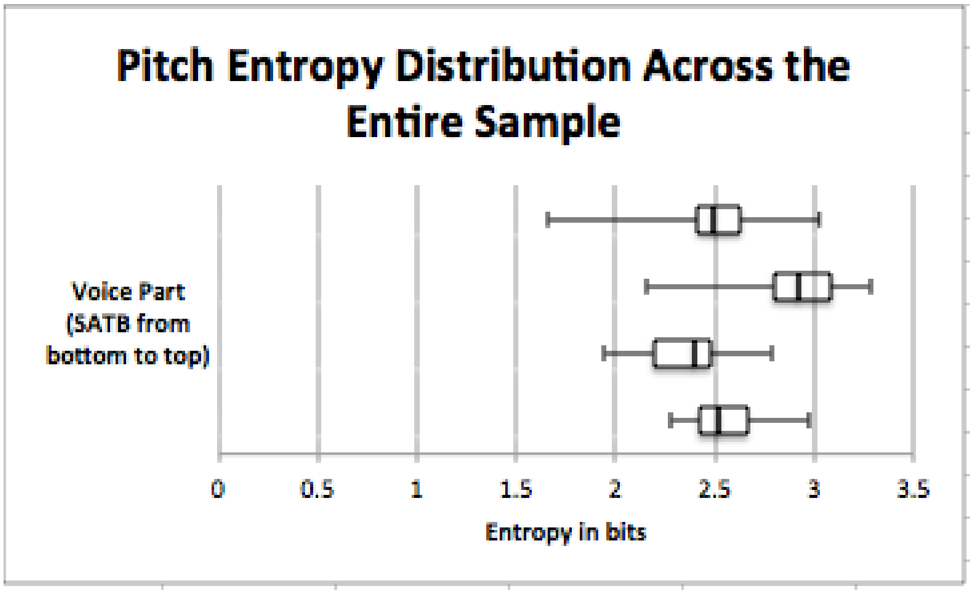
After the raw data was recorded (Appendix 1) for all 20 shape note tunes, data on pitch and rhythmic entropies was extracted for analysis. As mentioned in the **Methods**, the two sections of the Sacred Harp from which data for this study has been drawn contain tune with two distinct purposes. Part I contains music meant to be sung at church services or other places/times of worship, and Part II contains music written for educational purposes or for use in singing societies. Hence in this analysis, the purpose tunes from Part I will be referred to as "worship purpose" and the purpose tunes from Part II will be referred to as "educational purpose." Following that model, this paper will proceed to present pitch entropy data on the 20-tune sample as a whole, and then move on to present subsets of data on pitch entropies of the 10 tunes for each independent purpose. Data will be presented through a data table storing all of the entropy values collected from the raw data followed by a box and whisker plot visualizing the spread of the data and an ensuing discussion on its significance. The same process will be repeated with rhythmic entropies.

4.2 Pitch Entropy in the Sample as a Whole

Soprano	Alto	Tenor	Bass
2.3939	**N/A**	**2.799**	**2.2078**

2.4308	**N/A**	**2.9121**	**1.6558**
2.5755	**N/A**	**3.0596**	**2.4304**
2.373	**N/A**	**3.0534**	**2.542**
2.3869	**N/A**	**2.6381**	**2.4147**
2.6245	**N/A**	**2.5431**	**2.4528**
2.5098	**N/A**	**3.2398**	**2.1339**
2.2759	**N/A**	**2.1492**	**2.313**
2.5311	**N/A**	**2.9376**	**2.4098**
2.4434	**N/A**	**2.5729**	**2.4248**
2.3444	**N/A**	**2.7862**	**2.5323**
2.8791	**2.7893**	**3.285**	**2.6743**
2.5109	**2.3905**	**2.8775**	**2.4275**
2.5235	**1.9434**	**2.8874**	**3.0243**
2.9628	**2.4736**	**3.1906**	**2.7175**
2.9726	**N/A**	**2.9175**	**2.7408**
2.5796	**N/A**	**3.1513**	**2.5971**
2.7669	**N/A**	**2.9932**	**2.5763**
2.9273	**2.1903**	**2.9037**	**2.609**
2.4908	**N/A**	**3.1742**	**2.685**

***Table Containing pitch entropy values for all 20 tunes in all voice parts**



***Box and Whisker plot of pitch entropies across all 20 Shape Note Tunes selected**

As seen in the box and whisker plot above, the tenor voice parts across the entire sample tended to have the highest pitch entropies. This is self evident in the fact that the first quartile for the tenor voice parts (second plot from the top) sits at a higher entropy value than even the third quartile of data from the other three voice parts. It also correlates strongly with the fact that the tenor voice is typically the melody line in shape note music.

4.3 Pitch Entropy by Purpose

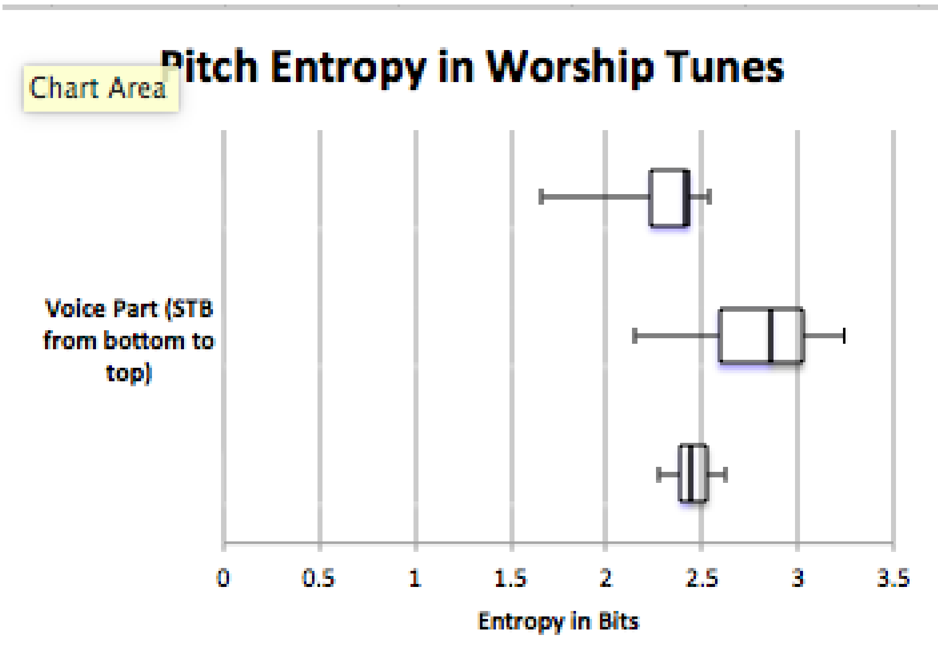
Name of Shape Note Tune	Pitch Entropy of Soprano Voice	Pitch Entropy of Alto Voice	Pitch Entropy of Tenor Voice	Pitch Entropy of Bass Voice
Missionary Hymn	**2.3939 bits**	**N/A**	**2.7990 bits**	**2.2078 bits**
Abbeville	**2.4308 bits**	**N/A**	**2.9121 bits**	**1.6558 bits**
The Pilgrim's Lot	**2.5755 bits**	**N/A**	**3.0596 bits**	**2.4304 bits**
Creation	**2.3730 bits**	**N/A**	**3.0534 bits**	**2.5420 bits**
Ball Hill	**2.3869 bits**	**N/A**	**2.6381 bits**	**2.4147 bits**
Mount Zion	**2.6245 bits**	**N/A**	**2.5431 bits**	**2.4528 bits**
Pleasant Grove	**2.5098 bits**	**N/A**	**3.2398 bits**	**2.1339 bits**
Animation	**2.2759 bits**	**N/A**	**2.1492 bits**	**2.3130 bits**
Essay	**2.5311 bits**	**N/A**	**2.9376 bits**	**2.4098 bits**
All is Well	**2.4434 bits**	**N/A**	**2.5729 bits**	**2.4248 bits**

***Pitch Entropy values for all voice parts in Worship Tunes**

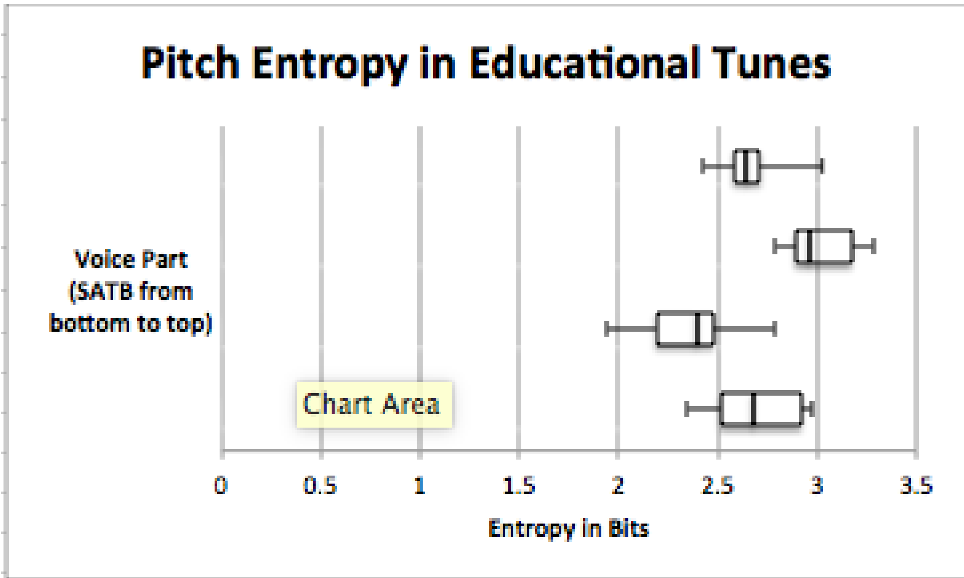
Name of Shape Note Tune	Pitch Entropy of Soprano Voice	Pitch Entropy of Alto Voice	Pitch Entropy of Tenor Voice	Pitch Entropy of Bass Voice
Spring	**2.3444 bits**	**N/A**	**2.7862 bits**	**2.5323 bits**
Alabama	**2.8791 bits**	**2.7893 bits**	**3.2850 bits**	**2.6743 bits**
EXIT	**2.5109 bits**	**2.3905 bits**	**2.8775 bits**	**2.4275 bits**
Newburgh	**2.5235 bits**	**1.9434 bits**	**2.8874 bits**	**3.0243 bits**
Mount Pleasant	**2.9628 bits**	**2.4736 bits**	**3.1906 bits**	**2.7175 bits**
Harmony	**2.9726 bits**	**N/A**	**2.9175 bits**	**2.7408 bits**

Star of Columbia	**2.5796 bits**	**N/A**	**3.1513 bits**	**2.5971 bits**
Family Bible	**2.7669 bits**	**N/A**	**2.9932 bits**	**2.5763 bits**
Huntington	**2.9273 bits**	**2.1903 bits**	**2.9037 bits**	**2.6090 bits**
Lena	**2.4908 bits**	**N/A**	**3.1742 bits**	**2.6850 bits**

***Pitch Entropy values for all voice parts in Educational Tunes**



***Pitch Entropy Box Plot for Worship tunes**



***Pitch Entropy Box Plot for Educational Tunes**

Based on the values recorded in table for Worship Tunes, it is apparent that for each voice part, pitch entropy values across all 10 worship form tunes are fairly close together. The column for Soprano voices best exemplifies this, where all 10 entropy values fall between 2.25 and 2.65 bits. However, there are a couple of notable outliers in the table as well, namely the pitch entropy of the bass voice in *Abbeville* and the tenor of *Animation*. In addition, was also a very notable trend in the tenor voice column, in that 9 out of the 10 tenor voices were the highest of the three pitch entropies for their tune. This is a trend we can expect to see when worship tune data is visualized in the next section, and correlates well with the fact that in shape note music with three voices (the case for all 10 worship form tunes), the tenor voice is generally considered to be the melody line, with the Soprano and Bass serving as harmony parts.

Drawing from the table for Educational Tunes, the pitch entropy values are again strikingly consistent within each vocal part. One of the main details setting its data apart from the values in the Worship Tunes table, however, is the distinctive lack of outliers in the Soprano, Tenor, and Bass parts. The only noteworthy outlier within its column appears to be the Alto line of *Alabama*. It is interesting to note that of the 557 shape note tunes in the 1991 edition, the outlier-generating tune *Alabama* ranks as the 68th most popular between 1995 and the present (fasola). That being said, the Tenor voice does lay claim to highest pitch entropy on its row 7 out of 10 times, with the exceptions being exclusively tunes written for four voices.

This information is only confirmed by the box plots, which show the tenor voice (second from the top on both plots) generally carrying the highest entropy values of all four voice parts for both purposes. Both plots also seem to show similar data values and distributions between the Soprano and Bass voice parts within each purpose. Between purposes, however, it would appear that entropy values for educational tunes trend slightly higher and vary somewhat less for the same voice part than worship tunes. The statistical mean entropy values from each voice part (across different tunes) seems to confirm this:

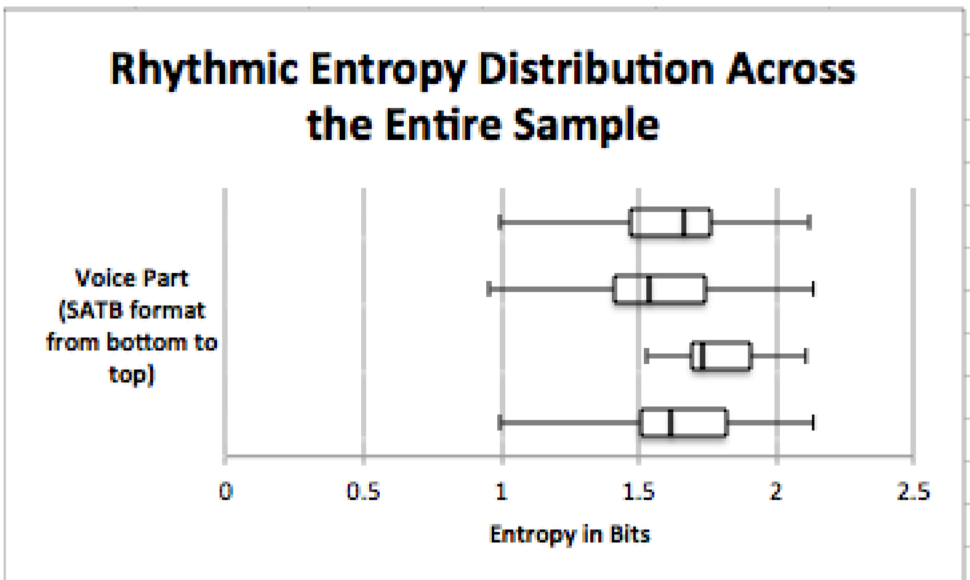
Mean (Worship)	**2.4545 bits**	**N/A**	**2.7905 bits**	**2.2989 bits**
Mean (Educational)	**2.6958 bits**	**2.3574 bits**	**3.0167 bits**	**2.6584 bits**
	Soprano	**Alto**	**Tenor**	**Bass**

4.4 Rhythmic Entropy in the Sample as a Whole

Soprano	Alto	Tenor	Bass
0.9955	**N/A**	**0.9509**	**0.9955**
1.524	**N/A**	**1.5891**	**1.1857**
1.4188	**N/A**	**1.1083**	**1.436**
1.6128	**N/A**	**1.7714**	**1.7102**
1.2095	**N/A**	**1.0856**	**1.1708**
2.1347	**N/A**	**2.1347**	**2.1229**
1.4591	**N/A**	**1.3735**	**1.4958**
1.5192	**N/A**	**1.3515**	**1.5716**
1.6153	**N/A**	**1.4908**	**1.6746**
1.7313	**N/A**	**1.8385**	**1.7457**

1.8704	N/A	1.7186	1.762
1.7986	1.5269	1.4748	1.781
1.8904	1.696	1.7315	1.6716
1.7697	1.9028	2.0306	1.6578
1.8362	2.1075	1.4724	2.1204
1.621	1.7305	1.64	1.4853
1.4173	N/A	1.4776	1.7736
1.5392	N/A	1.423	1.1469
1.83	N/A	1.8046	1.7566
1.8092	N/A	1.6984	1.5994

***Table Containing rhythmic entropy values for all 20 tunes in all voice parts**



***Rhythmic Entropy Box Plot for Full Sample**

As seen in the data above, rhythmic entropy values were strikingly consistent with one another across voice parts for the entire 20 tune sample, with the only notable exception being the alto voice (second from the bottom on the box plot), which has its lower extreme close to the median values for the other voice parts. That being said, the alto voice part is the one that the least data could be collected on, having only ¼ the number of tunes to be drawn from when compared to the others. Such a small sample size could mean that any difference that emerged in the alto parts did so by chance.

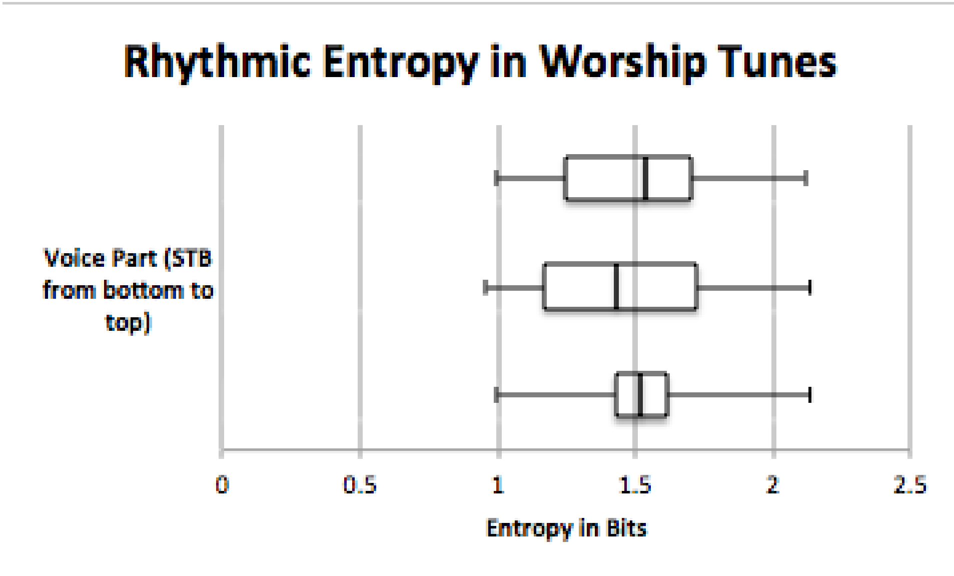
4.4 Rhythmic Entropy by Purpose

Name of Shape Note Tune	Rhythmic Entropy of Soprano Voice	Rhythmic Entropy of Alto Voice	Rhythmic Entropy of Tenor Voice	Rhythmic Entropy of Bass Voice
Missionary Hymn	**0 .9955 bits**	**N/A**	**0.9509 bits**	**0.9955 bits**
Abbeville	**1.5240 bits**	**N/A**	**1.5891 bits**	**1.1857 bits**
The Pilgrim's Lot	**1.4188 bits**	**N/A**	**1.1083 bits**	**1.4360 bits**
Creation	**1.6128 bits**	**N/A**	**1.7714 bits**	**1.7102 bits**
Ball Hill	**1.2095 bits**	**N/A**	**1.0856 bits**	**1.1708 bits**
Mount Zion	**2.1347 bits**	**N/A**	**2.1347 bits**	**2.1229 bits**
Pleasant Grove	**1.4591 bits**	**N/A**	**1.3735 bits**	**1.4958 bits**
Animation	**1.5192 bits**	**N/A**	**1.3515 bits**	**1.5716 bits**
Essay	**1.6153 bits**	**N/A**	**1.4908 bits**	**1.6746 bits**
All is Well	**1.7313 bits**	**N/A**	**1.8385 bits**	**1.7457 bits**

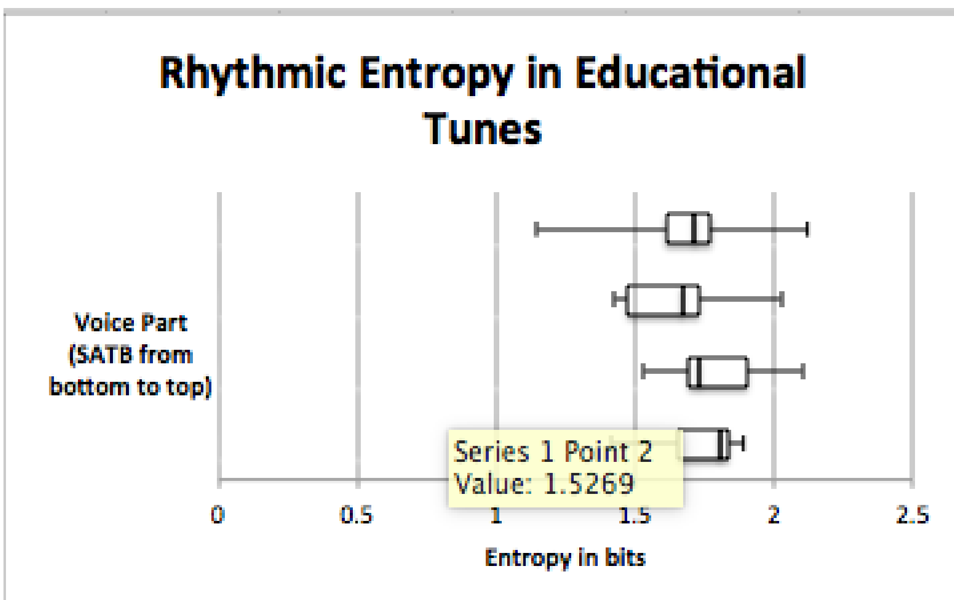
***Rhythmic Entropy values for all voice parts in Worship Tunes**

Name of Shape Note Work	Rhythmic Entropy of Soprano Voice	Rhythmic Entropy of Alto Voice	Rhythmic Entropy of Tenor Voice	Rhythmic Entropy of Bass Voice
Spring	**1.8704 bits**	**N/A**	**1.7186 bits**	**1.7620 bits**
Alabama	**1.7986 bits**	**1.5269 bits**	**1.4748 bits**	**1.7810 bits**
EXIT	**1.8904 bits**	**1.6960 bits**	**1.7315 bits**	**1.6716 bits**
Newburgh	**1.7697 bits**	**1.9028 bits**	**2.0306 bits**	**1.6578 bits**
Mount Pleasant	**1.8362 bits**	**2.1075 bits**	**1.4724 bits**	**2.1204 bits**
Harmony	**1.6210 bits**	**N/A**	**1.6400 bits**	**1.4853 bits**
Star of Columbia	**1.4173 bits**	**N/A**	**1.4776 bits**	**1.7736 bits**
Family Bible	**1.5392 bits**	**N/A**	**1.4230 bits**	**1.1469 bits**
Huntington	**1.8300 bits**	**1.7305 bits**	**1.8046 bits**	**1.7566 bits**
Lena	**1.8092 bits**	**N/A**	**1.6984 bits**	**1.5994 bits**

***Rhythmic Entropy values for all voice parts in Educational Tunes**



***Rhythmic Entropy Box Plot for Worship Tunes**



***Rhythmic Entropy Box Plot for Educational Tunes**

Much like its counterpart pitch entropy table, the values in the rhythmic entropy table for worship form are mostly consistent, but with a few notable outliers. Most notable among these are the tunes *Missionary Hymn* and *Mount Zion*. Missionary Hymn is a clear outlier on the low end of the rhythmic entropy spectrum, posting the only entropy values in any of the four data tables to be less than 1 bit. Mount Zion on the other hand, is an outlier on the upper end of the rhythmic entropy spectrum, posting the only rhythmic entropy values over 2 bits in worship form. It is also worth noting that the trend seen in pitch entropy for the tenor voice to have the highest value of its tune has not carried over to the rhythmic entropy values of the same tunes.

Not surprisingly, the rhythmic entropy table for educational form displays the general consistency as all of the others, coupled with the same lack of notable outliers within a column as seen in its pitch entropy counterpart. The only significant outlier occurs in the tenor Bass voice of *Family Bible*. Unlike *Alabama*, however, the outlier generating *Family Bible* does not rank very high on fasola's list of most popular tunes between 1995 and the present, instead ranking somewhat low at the 442nd spot. Given that the lack of outliers occurs in both the pitch and rhythmic entropy tables for educational tunes, it stands to reason that this may be indicative a trait shared among educational tunes. As with worship tunes though, they do not appear to have a single voice part that consistently produces the highest entropy value within a tune. It is interesting to note that of the shape note tunes analyzed in this study, 16 out of 20 have their highest pitch entropy value in the tenor voice (14 out of 15 tunes with 3 voices), while no such trend exists among rhythmic entropies.

Box and Whisker plots for rhythmic entropy show little variance between voice parts or between purposes. This could be indicative of a feature of shape note music, especially given that the mean values for rhythmic entropy across the same voice part in all tunes within each purpose are also so close together:

Mean (Worship)	**1.5220 bits**	**N/A**	**1.4694 bits**	**1.5109 bits**
Mean (Educational)	**1.7382 bits**	**1.7927 bits**	**1.6472 bits**	**1.6755 bits**

	Soprano	Alto	Tenor	Bass

4.5 Form and Popularity

Although each tune's form and popularity were considered as a part of this study, no correlations with a tune's could be proven nor disproven based on these metrics alone. This was mainly due to the problem of small sample size. While the sample may have been drawn intentionally in even numbers from sections of *The Sacred Harp* with slightly different purposes, the same did not turn out to be true for varying forms and popularity levels. Additionally, the popularity data on shape note tunes from fasola's website references tunes by the page they appear on in the 1991 revision that its data is drawn from, which differ slightly from those used in the 1860 version for this study. Because of this, popularity data on some tunes that otherwise have raw data associated with them is completely absent, having proved itself infeasible to track down in the 1991 edition.

5. Discussion

5.1 Reasoning From the Results

Based on the results of this study, there is evidence to support the idea that there are some trends across note-by-note pitch and rhythm entropy values both within and across both purposes for shape note music analyzed. Within educational purpose tunes, for instance, pitch and rhythmic entropy values for one voice part appear to be strikingly similar to those of the same voice part in other educational tunes, with a very low probability of outliers. The entropy values for worship form voice parts, by contrast, tend to have a broader range, with most voice parts still having similar entropy values to their counterparts in other worship tunes. Across both purposes, it seems that rhythmic entropy is approximately the same on average, in spite of worship outliers among voice parts for worship tunes. The same can be said of pitch entropy in the Tenor voice. On top of that, the tenor parts of shape note tunes with only three voices seem to have a their highest pitch entropy value in the Tenor voice regardless of form or purpose. This correlates strongly with the fact that the Tenor voice is the main melody in shape note music.

5.2 Limitations

One major limitation of this study was its scope. Although its methods were designed so that similar studies could easily be concocted through generalization, its results are not nearly as generalizable. Genre, for example, is a constant here, so no empirical conclusions can be drawn about genres outside of shape note music from the results of this study alone. Secondly, a sample size of just 20 shape note tunes from a single source is relatively small, and more data from more varied sources may need to be collected to cover the genre as a whole. Thirdly, this study does tie itself to a particular level of abstraction, specifically that of the note (as defined in the **Methods** section) and note sequence, meaning that it does not assess the entropy of the system of shape note music at any higher level of abstraction, such as that of the musical phrase or chord progression. This means that it cannot hope to paint a comprehensive picture of all entropy experienced at any level of abstraction from *The Sacred Harp*.

Lastly, this study faced limitations in terms of sample sizes when it came to form and popularity data. Form was limited from the beginning to exclude all anthems and odes, which are stored in Part III of The Sacred Harp. Popularity data was limited by the fact that the page numbers provided in the popularity dataset often failed to line up with the White and King version from 1860. Instead, they are drawn from a version of *The Sacred Harp* published in 1991. Alto voice data was also very scarce for this study, with only 5 of the 20 tunes covered having this voice part in the 1860 edition. As a result, no significant conclusions could be drawn from this data.

6. Conclusion

Over the course of this study, an objective analysis was conducted on elements of something that would normally be considered a highly subjective art form. In much the same the same way that humans and our subjective sensibilities interact with machines rooted in objective logic, the evidence now suggests that objective reasoning can be used to better understand the subjective art of music. More specifically, the findings from this study indicate that there may indeed be a relationship between information entropy and musical systems. This possibility is highlighted through a case study focused on the genre of shape note music as presented in one of the genre's most famous works, *The Sacred Harp*.

A couple of key trends emerged from the results of the study. For one, tenor voices tended to carry the highest entropy values when compared to others, and this held both within the same tunes and across different tunes both within each purpose and across all data in the sample. This correlates strongly with the fact that the melody in shape note music is typically carried by the tenor voice. Additionally, pitch entropy values across the entire sample were considerably higher than rhythmic ones, hinting at another potential characteristic of shape note music. Lastly, pitch entropy values for all four voice parts seem to vary slightly according to the tune's purpose, while the same cannot be said of rhythmic entropy values. All in all, the emergence of trends like these in the data may indicate that there is indeed a way to model musical systems objectively through the lens of information theory, and more specifically information entropy. More research would be required to confirm and test the generalizability of this study's findings, but the results can serve as a starting point for future research into the question of whether musical systems, and practices that emerged from subjective human sensibilities, can be modeled objectively via information theory.

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Appendix 1 -- Raw Data

<u>Piece #1:</u> *Missionary Hymn* **(Page 133, pos. ~~2~~)**

Soprano Voice Length in Notes: 54

(Pitch, Occurrence) Pairs: {(R, 1), (C5, 26), (A4, 6), (F4, 6), (F5, 4), (E5, 3), (D5, 6), (Bb4, 1), (C5**&**F5, 1)}

Pitch Entropy: 2.3939 bits

(Rhythm, Occurrence) Pairs: {(H, 9), (Q, 42), (DH, 2), (W, 1)**}**

Rhythmic Entropy: 0 .9955 bits

Tenor Voice Length in Notes: 57

(Pitch, Occurrence) Pairs: {(R, 1), (F4, 8), (A4, 13), (C5, 16), (D5, 5), (E4, 1), (Bb4, 5), (G4, 5), (F5, 2), (E5, 1)}

Pitch Entropy: 2.7990 bits

(Rhythm, Occurrence) Pairs: {(H, 11), (Q, 44), (GN, 1), (W, 1)**}**

Rhythmic Entropy: 0.9509 bits

Bass Voice Length in Notes: 54

Set of (Pitch, Occurrence) Pairs: {(R, 1), (F3, 31), (A3, 5), (Bb3, 3), (G3, 5), (C3, 4), (E3, 2), (G2, 1), (D3, 1), (C4, 1)}

Pitch Entropy: 2.2078 bits

Set of (Rhythm, Occurrence) Pairs: {(H, 9), (Q, 42), (DH, 2), (W, 1)}

Rhythmic Entropy: 0.9955 bits

Piece #2: *Abbeville* **(Page 33, pos. 2)**

Soprano Voice Length in Notes: 37

(Pitch, Occurrence) Pairs: {(R, 1), (D5, 14), (B4, 8), (E5, 3), (C5, 4), (G4, 3), (A4, 4)}

Pitch Entropy: 2.4308 bits

(Rhythm, Occurrence) Pairs: {(H, 14), (Q, 4), (E, 18), (DH, 1)}

Rhythmic Entropy: 1.5240 bits

Tenor Voice Length in Notes: 36

(Pitch, Occurrence) Pairs: {(R, 1), (G4, 8), (B4, 6), (A4, 3), (E4, 2), ($F^{\#}$4, 2), (D5, 6), (E5, 6), (G5, 2)}

Pitch Entropy: 2.9121 bits

(Rhythm, Occurrence) Pairs: {(H, 14), (Q, 5), (E, 16), (DH, 1)}

Rhythmic Entropy: 1.5891 bits

Bass Voice Length in Notes: 28

(Pitch, Occurrence) Pairs: {(R, 1), (G3, 13), (D3, 9), (E3, 5)}

Pitch Entropy: 1.6558 bits

(Rhythm, Occurrence) Pairs: {(H, 14), (Q, 13), (DH, 1)}

Rhythmic Entropy: 1.1857 bits

Piece #3: *The Pilgrim's Lot* **(Page 156, ~~pos. 2~~)**

Soprano Voice Length in Notes: 68

(Pitch, Occurrence) Pairs: {(R, 2), (G4, 13), (B4, 15), (D5, 20), (E5, 8), (A4, 7), (C5, 1), (G5, 1), (G4**&**D5, 1)}

Pitch Entropy: 2.5755 bits

(Rhythm, Occurrence) Pairs: {(E, 38), (Q, 24), (DQ, 4), (DH, 1), (DH**T**DQ, 1)}

Rhythmic Entropy: 1.4188 bits

Tenor Voice Length in Notes: 79

(Pitch, Occurrence) Pairs: {(R, 2), (D4, 7), (E4, 11), (G4, 22), (B4, 8), (C5, 7), (D5, 10), (A4, 5), (F$^{\#}$4, 1), (G5, 2), (E5, 4)}

Pitch Entropy: 3.0596 bits

(Rhythm, Occurrence) Pairs: {(E, 59), (Q, 15), (DH, 1), (DQ, 3), (DH**T**DQ, 1)}

Rhythmic Entropy: 1.1083 bits

Bass Voice Length in Notes: 67

(Pitch, Occurrence) Pairs: {(R, 2), (D3, 16), (E3, 22), (G3, 17), (B2, 2), (A3, 4), (D3**&**G3, 1), (F$^{\#}$3, 1), (B3, 1), (G2**&**G3, 1)}

Pitch Entropy: 2.4304 bits

(Rhythm, Occurrence) Pairs: {(E, 36), (Q, 25), (DQ, 4), (DH, 1), (DH**T**DQ, 1)}

Rhythmic Entropy: 1.4360 bits

<u>Piece #4:</u> *Creation* **(Page 115, ~~pos. 2~~)**

Soprano Voice Length in Notes: 57

(Pitch, Occurrence) Pairs: {(R, 1), (C5, 24), (A4, 6), (G4, 1), (B4, 3), (F5, 14), (E5, 2), (G5, 1), (D5, 5)}

Pitch Entropy: 2.3730 bits

(Rhythm, Occurrence) Pairs: {(H, 12), (Q, 35), (DQ, 4), (E, 4), (W, 2)}

Rhythmic Entropy: 1.6128 bits

Alto Voice Length in Notes: 58

(Pitch, Occurrence) Pairs: {(R, 1), (F4, 6), (G4, 4), (A4, 9), (C5, 12), (D5, 8), (E5, 5), (F5, 4), (B4, 8), (G5, 1)}

Pitch Entropy: 3.0534 bits

(Rhythm, Occurrence) Pairs: {(H, 12), (Q, 32), (DQ, 4), (E, 8), (W, 2)}

Rhythmic Entropy: 1.7714 bits

Tenor Voice Length in Notes: 57

(Pitch, Occurrence) Pairs: {(R, 1), (F3, 23), (C3, 11), (D3, 5), (E3, 3), (A3, 6), (G3, 5), (B^{b}3, 2), (B^{b}2, 1)}

Pitch Entropy: 2.5420 bits

(Rhythm, Occurrence) Pairs: {(H, 12), (Q, 33), (DQ, 4), (E, 6), (W, 2)}

Rhythmic Entropy: 1.7102 bits

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Piece #5: *Ball Hill* **(Page 118, pos. 1)**

Soprano Voice Length in Notes: 56

(Pitch, Occurrence) Pairs: {(E5, 18), (C5, 13), (D5, 6), (B4, 7), (A4, 10), (G4, 1), (R, 1)}

Pitch Entropy: 2.3869 bits

(Rhythm, Occurrence) Pairs: {(H, 14), (Q, 38), (DH, 3), (DW**T**DH, 1)}

Rhythmic Entropy: 1.2095 bits

Tenor Voice Length in Notes: 59

(Pitch, Occurrence) Pairs: {(A4, 16), (G4, 3), (B4, 9), (C5, 13), (D5, 6), (E5, 9), (E4, 2), (R, 1)}

Pitch Entropy: 2.6381 bits

(Rhythm, Occurrence) Pairs: {(H, 11), (Q, 44), (DH, 3), (DW**T**DH, 1)}

Rhythmic Entropy: 1.0856 bits

Bass Voice Length in Notes: 57

(Pitch, Occurrence) Pairs: {(E3, 21), (A3, 9), (G$^{\#}$3, 7), (F$^{\#}$3, 11), (D3, 5), (A2**&**A3, 3), (R, 1)}

Pitch Entropy: 2.4147 bits

(Rhythm, Occurrence) Pairs: {(H, 13), (Q, 40), (DH, 3), (DW**T**DH, 1)}

Rhythmic Entropy: 1.1708 bits

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<u>Piece #6:</u> *Mount Zion* **(Page 88, pos. 2) *522**

Soprano Voice Length in Notes: 43

(Pitch, Occurrence) Pairs: {(C5, 13), (F5, 13), (G5, 1), (A4, 3), (B^{b}4, 2), (D5, 4), (F5**&**C5, 3), (R, 3), (E5, 1)}

Pitch Entropy: 2.6245 bits

(Rhythm, Occurrence) Pairs: {(H, 9), (Q, 14), (E, 11), (DQ, 7), (W, 2)}

Rhythmic Entropy: 2.1347 bits

Tenor Voice Length in Notes: 43

(Pitch, Occurrence) Pairs: {(F4, 8), (A4, 5), (C5, 16), (D5, 5), (G4, 4), (R, 3), (F5, 1), (B^b4, 1)}

Pitch Entropy: 2.5431 bits

(Rhythm, Occurrence) Pairs: {(H, 9), (Q, 14), (E, 11), (DQ, 7), (W, 2)}

Rhythmic Entropy: 2.1347 bits

Bass Voice Length in Notes: 42

(Pitch, Occurrence) Pairs: {(F3, 20), (A3, 4), (C3, 6), (D3, 3), (E3, 1), (G3, 2), (F3**&**C3, 2), (R, 3), (C4, 1)}

Pitch Entropy: 2.4528 bits

(Rhythm, Occurrence) Pairs: {(H, 9), (Q, 15), (E, 9), (DQ, 7), (W, 2)}

Rhythmic Entropy: 2.1229 bits

Piece #7: *Pleasant Grove* **(Page 107, pos. 1)**

Soprano Voice Length in Notes: 54

(Pitch, Occurrence) Pairs: {(R, 3), (E5, 23), (C5, 10), (G5, 5), (D5, 5), (A5, 1), (B4, 4), (A4, 2), (A5**&**E5, 1)}

Pitch Entropy: 2.5098 bits

(Rhythm, Occurrence) Pairs: {(DH, 9), (H, 18), (Q, 27)}

Rhythmic Entropy: 1.4591 bits

Tenor Voice Length in Notes: 56

(Pitch, Occurrence) Pairs: {(R, 3), (A4, 10), (C5, 9), (B4, 7), (G4, 5), (E4, 2), (D5, 4), (E5, 7), (A5, 5), (G5, 3), (F5, 1)}

Pitch Entropy: 3.2398 bits

(Rhythm, Occurrence) Pairs: {(DH, 9), (H, 14), (Q, 33)}

Rhythmic Entropy: 1.3735 bits

Bass Voice Length in Notes: 51

(Pitch, Occurrence) Pairs: {(R, 3), (A3, 14), (E3, 23), (C3, 3), (G3, 5), (A2, 2), (D3, 1)}

Pitch Entropy: 2.1339 bits

(Rhythm, Occurrence) Pairs: {(DH, 9), (H, 21), (Q, 21)}

Rhythmic Entropy: 1.4958 bits

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Piece #8: *Animation* **(Page 103, pos. 1) *424**

Soprano Voice Length in Notes: 62

(Pitch, Occurrence) Pairs: {(R, 1), (G4, 7), (D5, 30), (G5, 3), (E5, 6), (B4, 8), (A4, 6), (C5, 1)}

Pitch Entropy: 2.2759 bits

(Rhythm, Occurrence) Pairs: {(DQ, 9), (Q, 20), (E, 32), (DH, 1)}

Rhythmic Entropy: 1.5192 bits

Tenor Voice Length in Notes: 68

(Pitch, Occurrence) Pairs: {(R, 1), (D4, 2), (G4, 19), (B4, 12), (A4, 8), (C5, 2), (D5, 12), (G5, 6), (E5, 5), ($F^{\#}$5, 1)}

Pitch Entropy: 2.1492 bits

(Rhythm, Occurrence) Pairs: {(DQ, 9), (Q, 14), (E, 44), (DH, 1)}

Rhythmic Entropy: 1.3515 bits

Bass Voice Length in Notes: 58

(Pitch, Occurrence) Pairs: {(R, 1), (G3, 29), (C3, 2), (D3, 10), (E3, 4), (A3, 6), (B3, 2), (C4, 2), (D4, 2)}

Pitch Entropy: 2.3130 bits

(Rhythm, Occurrence) Pairs: {(DQ, 9), (Q, 24), (E, 24), (DH, 1)}

Rhythmic Entropy: 1.5716 bits

<u>**Piece #9:**</u> *Essay* **(Page 157, pos. 1) *247**

Soprano Voice Length in Notes: 58

(Pitch, Occurrence) Pairs: {(R, 2), (A4, 8), (D5, 20), ($F^{\#}5$, 7), (B4, 7), ($C^{\#}5$, 4), (E5, 10)}

Pitch Entropy: 2.5311 bits

(Rhythm, Occurrence) Pairs: {(DQ, 8), (Q, 24), (E, 24), (DH, 2)}

Rhythmic Entropy: 1.6153 bits

Tenor Voice Length in Notes: 64

(Pitch, Occurrence) Pairs: {(R, 2), ($F^{\#}4$, 10), (A4, 20), (D5, 10), ($F^{\#}5$, 6), (E5, 5), (G4, 2), (E4, 2), (B4, 4), ($C^{\#}5$, 2), (G5, 1)}

Pitch Entropy: 2.9376 bits

(Rhythm, Occurrence) Pairs: {(DQ, 6), (Q, 22), (E, 34), (DH, 2)}

Rhythmic Entropy: 1.4908 bits

Bass Voice Length in Notes: 56

(Pitch, Occurrence) Pairs: {(R, 2), (D3, 17), (F$^{\#}$3, 12), (G3, 7), (A3, 13), (E3, 4), (B3, 1)}

Pitch Entropy: 2.4098 bits

(Rhythm, Occurrence) Pairs: {(DQ, 10), (Q, 22), (E, 22), (DH, 2)}

Rhythmic Entropy: 1.6746 bits

<u>Piece #10:</u> *All is Well* **(Page 122, pos. 1) *129**

Soprano Voice Length in Notes: 50

(Pitch, Occurrence) Pairs: {(A4, 8), (E5, 20), (D5, 1), (C$^{\#}$5, 10), (F$^{\#}$5, 5), (B4**&**E5, 1), (C$^{\#}$5**&**F$^{\#}$5, 1), (C$^{\#}$5**&**E5, 1), (B4, 3)}

Pitch Entropy: 2.4434 bits

(Rhythm, Occurrence) Pairs: {(H, 12), (Q, 28), (E, 4), (DQ, 4), (W, 2)}

Rhythmic Entropy: 1.7313 bits

Tenor Voice Length in Notes: 55

(Pitch, Occurrence) Pairs: {(A4, 14), (G$^{\#}$4, 6), (B4, 9), (C$^{\#}$5, 13), (D5, 6), (E5, 6), (E4, 1)}

Pitch Entropy: 2.5729 bits

(Rhythm, Occurrence) Pairs: {(H, 7), (Q, 30), (E, 8), (DQ, 8), (W, 2)}

Rhythmic Entropy: 1.8385 bits

Bass Voice Length in Notes: 52

(Pitch, Occurrence) Pairs: {(A3, 11), (E3, 21), (D3, 3), (C$^{\#}$3, 3), (F$^{\#}$3, 9), (A2, 1), (G$^{\#}$3**&**B3, 1), (G$^{\#}$3, 2), (A2**&**A3, 1)}

Pitch Entropy: 2.4248 bits

(Rhythm, Occurrence) Pairs: {(H, 10), (Q, 30), (E, 5), (DQ, 5), (W, 2)}

Rhythmic Entropy: 1.7457 bits

<u>Piece #11:</u> *Spring* **(Page 188 – 189, pos. 1) *489**

Soprano Voice Length in Notes: 113

(Pitch, Occurrence) Pairs: {(R, 1), (D5, 16), (B4, 30), (C5, 13), (E5, 4), (A4, 17), (G4, 19), ($F^{\#}$4, 10), (E4, 1), (D4, 2)}

Pitch Entropy: 2.3444 bits

(Rhythm, Occurrence) Pairs: {(Q, 16), (Trip, 24), (E, 61), (DQ, 4), (H, 1), (S, 6), (DH, 1)}

Rhythmic Entropy: 1.8704 bits

Tenor Voice Length in Notes: 117

(Pitch, Occurrence) Pairs: {(R, 1), (D4, 2), (G4, 24), (A4, 21), (B4, 28), (C5, 15), (D5, 16), ($F^{\#}$4, 5), (E5, 3), ($F^{\#}$5, 1), (G5, 1)}

Pitch Entropy: 2.7862 bits

(Rhythm, Occurrence) Pairs: {(Q, 15), (Trip, 27), (E, 59), (DQ, 4), (H, 1), (S, 10), (DH, 1)}

Rhythmic Entropy: 1.7186 bits

Bass Voice Length in Notes: 74

(Pitch, Occurrence) Pairs: {(R, 1), (D3, 11), (G3, 30), (A3, 9), (B3, 11), (E3, 6), ($F^{\#}$3, 3), (C4, 1), (D4, 1), (C3, 1)}

Pitch Entropy: 2.5323 bits

(Rhythm, Occurrence) Pairs: {(Q, 37), (E, 25), (DQ, 6), (Trip, 3), (H, 1), (DQ**T**Q, 1), (DH, 1)}

Rhythmic Entropy: 1.7620 bits

<u>Piece #12:</u> *Alabama* **(Page 196, pos. 1) *68**

Soprano Voice Length in Notes: 62

(Pitch, Occurrence) Pairs: {(R, 2), (B4, 19), (G4, 4), (A4, 10), (E5, 10), (D5, 8), (C5, 1), (F$^{\#}$5, 3), (G5, 3), (F$^{\#}$4, 1), (E4, 1)}

Pitch Entropy: 2.8791 bits

(Rhythm, Occurrence) Pairs: {(DQ, 8), (Q, 20), (E, 30), (DH, 1), (W, 1), (DW**T**DQ**T**Q, 1), (DH**T**DQ, 1)}

Rhythmic Entropy: 1.7986 bits

Alto Voice Length in Notes: 65

(Pitch, Occurrence) Pairs: {(R, 2), (B4, 19), (D5, 12), (E5, 14), (G4, 5), (A4, 6), (F$^{\#}$5, 2), (G5, 1), (C5, 2), (F$^{\#}$4, 1), (E4, 1)}

Pitch Entropy: 2.7893 bits

(Rhythm, Occurrence) Pairs: {(DQ, 7), (Q, 20), (E, 36), (DH, 1), (5W, 1)}

Rhythmic Entropy: 1.5269 bits

Tenor Voice Length in Notes: 69

(Pitch, Occurrence) Pairs: {(R, 2), (E4, 5), (G4, 8), (A4, 8), (B4, 11), (C5, 6), (D5, 5), (E5, 12), (F$^{\#}$4, 3), (F$^{\#}$5, 6), (G5, 3)}

Pitch Entropy: 3.2850 bits

(Rhythm, Occurrence) Pairs: {(DQ, 6), (Q, 16), (E, 44), (DH, 1), (3W, 1), (DW**T**DQ, 1)}

Rhythmic Entropy: 1.4748 bits

Bass Voice Number of Notes: 59

(Pitch, Occurrence) Pairs: {(R, 1), (E3, 19), (G3, 9), (D3, 6), ($F^{\#}$3, 5), (B2, 3), (A3, 7), (B3, 9)}

Pitch Entropy: 2.6743 bits

(Rhythm, Occurrence) Pairs: {(DQ, 6), (Q, 22), (E, 27), (DH, 1), (DQ**T**Q, 1), (DW, 1), (DW**T**DH**T**DQ, 1)}

Rhythmic Entropy: 1.7810 bits

<u>**Piece #13:**</u> *EXIT* **(Page 181, pos. 1) *306**

Soprano Voice Length in Notes: 49

(Pitch, Occurrence) Pairs: {(B4, 20), (G4, 4), (A4, 6), (E5, 9), (D5, 5), (C5, 2), (R, 1), ($F^{\#}$5, 1), (E4, 1)}

Pitch Entropy: 2.5109 bits

(Rhythm, Occurrence) Pairs: {(H, 2), (Q, 30), (DQ, 1), (E, 9), (DH, 1), (W, 3), (2W, 1), (DE, 1), (S, 1)}

Rhythmic Entropy: 1.8904 bits

Alto Voice Length in Notes: 55

(Pitch, Occurrence) Pairs: {(E4, 9), (G4, 9), (B4, 19), (A4, 10), ($F^{\#}$4, 6), (D4, 1), (R, 1)}

Pitch Entropy: 2.3905 bits

(Rhythm, Occurrence) Pairs: {(H, 3), (Q, 29), (DH, 2), (E, 18), (W, 2), (DW, 1)}

Rhythmic Entropy: 1.6960 bits

Tenor Voice Length in Notes: 61

(Pitch, Occurrence) Pairs: {(B4, 17), (G4, 4), (E4, 2), (F$^{\#}$4, 1), (C5, 5), (A4, 2), (E5, 14), (D5, 7), (R, 1), (F$^{\#}$5, 5), (G5, 3)}

Pitch Entropy: 2.8775 bits

(Rhythm, Occurrence) Pairs: {(H, 3), (Q, 28), (DQ, 2), (E, 24), (DH, 1), (W, 3)}

Rhythmic Entropy: 1.7315 bits

Bass Voice Number of Notes: 56

(Pitch, Occurrence) Pairs: {(E3, 21), (B3, 10), (A3, 9), (G3, 9), (B2, 3), (R, 1), (F$^{\#}$3, 2), (D3, 1)}

Pitch Entropy: 2.4275 bits

(Rhythm, Occurrence) Pairs: {(H, 5), (Q, 34), (DQ, 2), (E, 12), (DH, 1), (W, 2)}

Rhythmic Entropy: 1.6716 bits

<u>Piece #14:</u> *Newburgh* **(Page 182, pos. 1) *113**

Soprano Voice Length in Notes: 66

(Pitch, Occurrence) Pairs: {(C5, 18), (E5, 18), (D5, 16), (A4, 3), (R, 2), (G5, 3), (B4, 2), (G4, 3), (E4, 1)}

Pitch Entropy: 2.5235 bits

(Rhythm, Occurrence) Pairs: {(W, 3), (Q, 44), (DH, 7), (3W, 1), (H, 2), (2W**T**DH, 1), (DE, 1), (S, 1), (E, 6)}

Rhythmic Entropy: 1.7697 bits

Alto Voice Length in Notes: 53

(Pitch, Occurrence) Pairs: {(E4, 8), (G4, 28), (A4, 10), (F4, 2), (R, 3), (D4, 2)}

Pitch Entropy: 1.9434 bits

(Rhythm, Occurrence) Pairs: {(W, 3), (Q, 33), (DH, 6), (H, 4), (2W, 1), (E, 4), (2W**T**DH, 1), (4W, 1)}

Rhythmic Entropy: 1.9028 bits

Tenor Voice Length in Notes: 70

(Pitch, Occurrence) Pairs: {(G4, 7), (E4, 2), (C5, 20), (D5, 15), (E5, 10), (B4, 3), (A4, 6), (R, 3), (F4, 1), (F5, 2), (G5, 1)}

Pitch Entropy: 2.8874 bits

(Rhythm, Occurrence) Pairs: {(W, 4), (Q, 38), (DH, 4), (E, 17), (DE, 1), (S, 1), (DQ, 1), (H, 2), (2W**T**DH, 1), (4W, 1)}

Rhythmic Entropy: 2.0306 bits

Bass Voice Length in Notes: 72

(Pitch, Occurrence) Pairs: {(C3, 10), (E3, 7), (G3, 20), (C4, 11), (A3, 8), (D3, 5), (F3, 2), (C3**&**C4, 3), (D3**&**D4, 1), (B3, 3), (R, 2)}

Pitch Entropy: 3.0243 bits

(Rhythm, Occurrence) Pairs: {(W, 3), (Q, 48), (DH, 6), (E, 9), (DQ, 1), (H, 4), (4W, 1)}

Rhythmic Entropy: 1.6578 bits

Piece #15: *Mount Pleasant* **(Page 219, ~~pos. 2~~) *181**

Soprano Voice Length in Notes: 52

(Pitch, Occurrence) Pairs: {(C5, 11), (D5, 6), (A4, 12), (G4, 6), (R, 4), (B^{b}4, 6), (F5, 3), (E5, 1), (F4, 2), (A4&C5, 1)}

Pitch Entropy: 2.9628 bits

(Rhythm, Occurrence) Pairs: {(Q, 21), (DH, 3), (E, 20), (H, 6), (W, 2)}

Rhythmic Entropy: 1.8362 bits

Alto Voice Length in Notes: 44

(Pitch, Occurrence) Pairs: {(A4, 11), (R, 2), (F4, 10), (G4, 13), (B^{b}4, 4), (C4, 2), (D4, 1), (E4, 1)}

Pitch Entropy: 2.4736 bits

(Rhythm, Occurrence) Pairs: {(DH, 2), (Q, 17), (E, 16), (DQ, 2), (H, 4), (W, 2), (2WTH, 1)}

Rhythmic Entropy: 2.1075 bits

Tenor Voice Length in Notes: 64

(Pitch, Occurrence) Pairs: {(F5, 8), (R, 6), (E5, 5), (D5, 8), (C5, 11), (B^{b}4, 6), (A4, 8), (G4, 6), (F4, 5), (G5, 1)}

Pitch Entropy: 3.1906 bits

(Rhythm, Occurrence) Pairs: {(DH, 3), (Q, 24), (E, 34), (W, 2), (H, 1)}

Rhythmic Entropy: 1.4724 bits

Bass Voice Length in Notes: 43

(Pitch, Occurrence) Pairs: {(F3, 15), (B^{b}3, 2), (A3, 4), (G3, 5), (R, 4), (C3, 6), (D3, 3), (E3, 4)}

Pitch Entropy: 2.7175 bits

(Rhythm, Occurrence) Pairs: {(Q, 20), (DE, 1), (S, 1), (E, 12), (H, 4), (DW, 2), (W, 2), (DH, 1)}

Rhythmic Entropy: 2.1204 bits

<u>Piece #16:</u> *Harmony* **(Page 172, pos. 1) *168**

Soprano Voice Length in Notes: 82

(Pitch, Occurrence) Pairs: {(R, 2), (B4, 19), (D5, 15), (C5, 9), (A4, 10), (G4, 10), (F$^{\#}$4, 1), (E5, 9), (F$^{\#}$5, 3), (G5, 4)}

Pitch Entropy: 2.9726 bits

(Rhythm, Occurrence) Pairs: {(H, 4), (Q, 38), (E, 34), (DH, 4), (2W, 1), (W, 1)}

Rhythmic Entropy: 1.6210 bits

Tenor Voice Length in Notes: 81

(Pitch, Occurrence) Pairs: {(R, 3), (D5, 21), (B4, 16), (A4, 7), (G4, 9), (D4, 1), (C5, 8), (E5, 9), (F$^{\#}$5, 1), (G5, 6)}

Pitch Entropy: 2.9175 bits

(Rhythm, Occurrence) Pairs: {(H, 3), (Q, 43), (E, 28), (DH, 4), (2W, 1), (DW, 1), (W, 1)}

Rhythmic Entropy: 1.6400 bits

Bass Voice Length in Notes: 67

(Pitch, Occurrence) Pairs: {(R, 1), (G3, 22), (A3, 3), (F$^{\#}$3, 4), (E3, 14), (D3, 12), (B2, 2), (C3, 2), (B3, 5), (D4, 1), (C4, 1)}

Pitch Entropy: 2.7408 bits

(Rhythm, Occurrence) Pairs: {(H, 4), (Q, 46), (DH, 5), (E, 10), (2W**T**Q, 1), (W, 1)}

Rhythmic Entropy: 1.4853 bits

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<u>Piece #17:</u> *Star of Columbia* **(Page 198 - 199, ~~pos. 2~~)**

Soprano Voice Length in Notes: 111

(Pitch, Occurrence) Pairs: {(R, 2), (E5, 34), (D5, 28), (C5, 16), (G5, 16), (F5, 2), (A5, 4), (B4, 6), (A4, 3)}

Pitch Entropy: 2.5796 bits

(Rhythm, Occurrence) Pairs: {(Q, 19), (E, 76), (DE, 7), (S, 7), (H, 2)}

Rhythmic Entropy: 1.4173 bits

Tenor Voice Length in Notes: 115

(Pitch, Occurrence) Pairs: {(R, 2), (C5, 12), (B4, 6), (A4, 21), (G4, 15), (E4, 12), (D5, 12), (E5, 15), (G5, 14), (A5, 6)}

Pitch Entropy: 3.1513 bits

(Rhythm, Occurrence) Pairs: {(Q, 15), (E, 78), (DE, 10), (S, 10), (H, 2)}

Rhythmic Entropy: 1.4776 bits

Bass Voice Length in Notes: 99

(Pitch, Occurrence) Pairs: {(R, 2), (A3, 24), (G3, 17), (E3, 32), (C3, 7), (D3, 7), (C4, 5), (B3, 3), (A2, 2)}

Pitch Entropy: 2.5971 bits

(Rhythm, Occurrence) Pairs: {(Q, 31), (E, 48), (DE, 9), (S, 9), (H, 2)}

Rhythmic Entropy: 1.7736 bits

<u>Piece #18:</u> *Family Bible* **(Page 165 - 166, ~~pos. 2~~) *442**

Soprano Voice Length in Notes: 103

(Pitch, Occurrence) Pairs: {(R, 2), (E5, 22), (C5, 18), (B4, 14), (A4, 21), (G4, 15), (D5, 7), (E4, 3), (G5, 1)}

Pitch Entropy: 2.7669 bits

(Rhythm, Occurrence) Pairs: {(H, 27), (Q, 63), (DQ, 5), (E, 5), (W, 2), (DH, 1)}

Rhythmic Entropy: 1.5392 bits

Tenor Voice Length in Notes: 105

(Pitch, Occurrence) Pairs: {(R, 2), (E4, 13), (A4, 20), (C5, 16), (D5, 10), (E5, 11), (F5, 5), (B4, 13), (G4, 10), (F4, 2), (G5, 2), (A5, 1)}

Pitch Entropy: 2.9932 bits

(Rhythm, Occurrence) Pairs: {(H, 25), (Q, 69), (DQ, 4), (E, 4), (W, 2), (DH, 1)}

Rhythmic Entropy: 1.4230 bits

Bass Voice Length in Notes: 96

(Pitch, Occurrence) Pairs: {(R, 2), (A3, 17), (E3, 38), (C3, 9), (D3, 9), (G3, 13), (A2, 2), (A2**&**A3, 3), (C4, 2), (B3, 1)}

Pitch Entropy: 2.5763 bits

(Rhythm, Occurrence) Pairs: {(H, 34), (Q, 59), (W, 2), (DH, 1)}

Rhythmic Entropy: 1.1469 bits

<u>**Piece #19:**</u> *Huntington* **(Page 193 - 194, pos. 2) *327**

Soprano Voice Length in Notes: 97

(Pitch, Occurrence) Pairs: {(A4, 16), (C$^{\#}$5, 19), (B4, 20), (E5, 18), (D5, 9), (G$^{\#}$4, 3), (F$^{\#}$5, 3), (R, 2), (F$^{\#}$4, 2), (E4, 4), (G$^{\#}$5, 1)}

Pitch Entropy: 2.9273 bits

(Rhythm, Occurrence) Pairs: {(W, 2), (DQ, 5), (E, 29), (Q, 50), (DH, 8), (2W, 1), (H, 2)}

Rhythmic Entropy: 1.8300 bits

Alto Voice Length in Notes: 81

(Pitch, Occurrence) Pairs: {(E4, 27), (A4, 22), (G$^{\#}$4, 17), (B4, 5), (F$^{\#}$4, 9), (R, 1)}

Pitch Entropy: 2.1903 bits

(Rhythm, Occurrence) Pairs: {(W, 3), (DH, 11), (Q, 47), (H, 1), (E, 17), (3W, 1), (DQ, 1)}

Rhythmic Entropy: 1.7305 bits

Tenor Voice Length in Notes: 106

(Pitch, Occurrence) Pairs: {(E4, 6), (A4, 20), (B4, 17), (C$^{\#}$5, 26), (E5, 14), (G$^{\#}$4, 5), (D5, 11), (F$^{\#}$5, 3), (R, 2), (F$^{\#}$4, 2)}

Pitch Entropy: 2.9037 bits

(Rhythm, Occurrence) Pairs: {(W, 3), (DQ, 6), (E, 40), (Q, 49), (DH, 5), (H, 2), (W**T**DH, 1)}

Rhythmic Entropy: 1.8046 bits

Bass Voice Length in Notes: 84

(Pitch, Occurrence) Pairs: {(A2, 10), (E3, 31), (F$^{\#}$3, 13), (A3, 12), (C$^{\#}$3, 5), (D3, 4), (G$^{\#}$3, 6), (B3, 3)}

Pitch Entropy: 2.6090 bits

(Rhythm, Occurrence) Pairs: {(W, 2), (DH, 10), (Q, 49), (H, 2), (E, 18), (H**T**2W**T**DH, 1), (DQ, 2)}

Rhythmic Entropy: 1.7566 bits

<u>**Piece #20:**</u> *Lena* **(Page 210, ~~pos. 2~~) *401**

Soprano Voice Length in Notes: 64

(Pitch, Occurrence) Pairs: {(C#5, 26), (B4, 10), (E5, 8), (R, 7), (F#5, 2), (E#5, 1), (D5, 4), (A4, 6)}

Pitch Entropy: 2.4908 bits

(Rhythm, Occurrence) Pairs: {(Q, 25), (DQ, 4), (E, 28), (DE, 3), (S, 3), (W, 1)}

Rhythmic Entropy: 1.8092 bits

Tenor Voice Length in Notes: 64

(Pitch, Occurrence) Pairs: {(F#4, 6), (G#4, 9), (A4, 9), (B4, 8), (C#5, 12), (R, 7), (E#5, 2), (F#5, 5), (D5, 4), (E4, 1), (E5, 1)}

Pitch Entropy: 3.1742 bits

(Rhythm, Occurrence) Pairs: {(Q, 25), (DQ, 4), (E, 30), (DE, 2), (S, 2), (W, 1)}

Rhythmic Entropy: 1.6984 bits

Bass Voice Length in Notes: 56

(Pitch, Occurrence) Pairs: {(F#3, 18), (C#3, 12), (E#3, 1), (G#3, 6), (R, 7), (B3, 2), (A3, 5), (C#4, 1), (E3, 4)}

Pitch Entropy: 2.6850 bits

(Rhythm, Occurrence) Pairs: {(Q, 25), (DQ, 6), (E, 23), (H, 1), (W, 1)}

Rhythmic Entropy: 1.5994 bits

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Appendix 2 – Tune Selection Code

This section of the Appendix contains the code that was run to select each of the 20 shape note tunes for analysis. For the first 10 selections, the value 1 to the scanner object when the program renders the prompt. For the next 10, a value of 2 is entered instead. This ensures that 10 tunes will be selected from part I, and 10 from part II of *The Sacred Harp*. Line numbers on the left hand side correspond to lines of code as they appeared in the original JGRASP file from which this code was copied. Parts III and IV (The book's Appendix) exist in *The Sacred Harp* and are provided for in the code, but were not analyzed in this study due to time constraints.

```
1 import java.util.Scanner;
2 import java.util.Random;
3
4 public class SacredHarpSongSelector {
5
6    public static void main (String [] args) {
7       Scanner scan = new Scanner(System.in);
8       Random rand = new Random();
9       int part = 0; //Indicates the (form-based) section
10      int page = 0;//page # within The Sacred harp
11      int position = 0;//Sometimes there is more than 1 piece on a page. If there is only one this value can be ignored.
12      int start = 0; //First page of the part
```

```
    int length = 0; //Length of the part (last page - start)

    //Prompt user to enter the number of the part they wish to select a piece from
     System.out.println("Enter the part (numerically) to select a song from (Appendix = 4):");
      part = scan.nextInt();

      //Defines the page range from which the piece can be selected
      if(part == 1){

         start = 27;
         length = 136;

      }else if(part == 2){

         start = 163;
         length = 62;

      }else if(part == 3){

```

```
            start = 225;
            length = 38;

        }else if(part == 4){

            start = 263;
            length = 135;

        }else{
            System.out.println("Error: not a valid section.");
        }

        //Pseudorandomly select page (from the specified range) and position on page
        page = rand.nextInt(length) + start;
        position = rand.nextInt(2) + 1;
        System.out.println("Selection: Page " + page + ", position " + position);
    }
}
```