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Influence of wall vibrations on the sound of brass wind instruments

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Influence of wall vibrations on the sound of brass wind instruments

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The results of an experimental and theoretical investigation of the influence of wall vibrations on the sound of brass wind instruments are presented. Measurements of the transmission function and input impedance of a trumpet, with the bell both heavily damped and freely vibrating, are shown to be consistent with a theory that assumes that the internal pressure causes an oscillation of the diameter of the pipe enclosing the air column. These effects are shown to be most significant in sections where there are flaring walls, which explains why damping these vibrations in cylindrical pipes normally produces no measurable effects.
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I. INTRODUCTION

The question of whether the material and workmanship of wind instruments in general, and brass wind instruments in particular, can or does significantly influence the radiated sound of those instruments was asked hundreds of years ago. During the ensuing generations of musicians, instrument makers and scientists, many were convinced of being in exclusive possession of the correct answer to this question and blaming the other side for simple-mindedness, closed-mindedness or jaundice.

Hermann Helmholtz dealt with this question in his legendary book *Die Lehre von den Tonempfindungen*, noting that:[1]

> Air vibrations in these instruments are most powerful and only hard, smooth tubes without leaks can fully resist these forces in order not to lose any of their original power.

A good summary of the diverse results of several scientists and instrument makers trying to determine the effect of wall material on the radiated sound of various wind instruments, in particular flue pipes between 1817 and 1940, has been published by Boner and Newman.[2] Of particular note is Miller's idea in 1909 to construct a double-wall flue pipe which can be filled with water during an experiment.[3] Boner and Newman themselves did observe some changes in the amplitudes of partials up to about 3 dB, but in their conclusions they attributed these effects to small geometric differences of the pipes under investigation. Lottermoser and Meyer repeated this experiment in 1962;[4] however, unlike Boner and Newman, they attributed the differences to the wall material without being able to give a convincing evidence for their claim.

In the mid-1960s Backus and Hundley published the results of important theoretical and experimental work concerning the effects of breathing modes of a pipe.[5–7] Their theoretical investigation of breathing modes showed a dependence of the speed of sound on a pressure induced oscillation of the cross-sectional area. They therefore confirmed observations of Savart and Liskovius that vibrating walls lower the pitch.[8,9] From their results, Backus and Hundley concluded that the relationship between the intensity level of the sound radiated by the vibrating walls L_{wall} and that radiated from the open end L_{mouth} is a function of the relative shift in frequency $\Delta f/f$ and is given by

$$L_{wall} - L_{mouth} = 20 \log \frac{2\Delta f}{f}. \quad (1)$$

With reasonable assumptions for typical organ pipes, this results in wall effects being about 40 dB below the main sound intensity. Backus and Hundley also repeated Miller's experiments without observing any significant change in harmonic amplitudes. They found that occasional changes in pitch and amplitude can be attributed to the degree the wall yields to the internal sound pressure, but this is only possible for elliptical or other non-cylindrical cross-sections. For perfectly cylindrical cross-sections they conclude "…*that the wall vibrations in organ pipes as commonly constructed have negligible influence on the steady pipe tone, and probably little on the transient buildup as well.*"

Ten years later Wogram investigated the wall vibrations of trombones.[10] He asked musicians to play the instruments and had them assess the subjective quality of the sound. He also made objective measurements using artificial lips. Wogram did observe spectral differences of up to 3 dB for different wall materials, however, he claimed that the variations in sound were almost indistinguishable. Likewise, although there were some interesting correlations between subjective assessments and objective measurements, he

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attributed these to differences in the response of the instrument.

In 1978 Richard Smith expressed the opinion that the material does not change the timbre or response, but wall thickness does.[11] According to his observations on trumpets at that time, he found the difference between two brass bells, one 0.5 mm thick and the other 0.3 mm thick, quite noticeable.

Smith determined that vibration amplitude is inversely proportional to the fourth power of wall thickness and he published beautiful holograms of bell eigenmodes, which were subsequently published in *Nature* approximately ten years later.[12] His conclusion in 1978 was that "…*material vibration appears to accentuate the higher frequencies and increase the responsiveness of the upper register.*"

Soon afterwards Lawson published the results of experiments with French horns where similar bells made of different alloys, annealed as well as unannealed, were compared.[13] Although the observed spectral differences did not exceed 3 dB, he concluded in a later article that "*The results of this experiment demonstrates unequivocally that the effect of the bell material vibration on the radiated sound is exceptionally strong.*"[14]

However, skeptical readers noted that since human players were used in the experiments it was possible that the spectral differences could be due to variations in embouchure, which even a professional player might not be able to avoid. Watkinson and Bowsher strengthened the skeptics' arguments by presenting results of finite element modeling, taking the position that wall vibrations should not have any noticeable effect.[15]

While more articles were published supporting the hypothesis that wall vibrations do affect the radiated sound of brass wind instruments and organ pipes (e.g., Pyle on the effect of lacquer and silver plating on horn tone[16]), Wogram and Smith became more skeptical. Within a decade after his original work, Wogram indicated that it requires a human player in order to modify the timbre according to what he senses with his hands or what he perceives as a near-field sound radiation, which does not propagate to the audience.[17] Smith again reported small but clearly measurable differences in the harmonics which were close to structural resonances, but his test subjects failed to detect these differences.[18] Therefore, he concluded that:

> …bell thickness does have a significant effect on the sound spectra measured at the player's ear position due to some sound radiation from the material itself. However, under controlled conditions players seem unable to distinguish between thick and thin materials.

At this point, however, Pyle was still strongly convinced of the effect that different materials and thicknesses had on the tone quality and responsiveness of French horns, but again his experiments suffered from the fact that human players had been used.[19]

Work on the vibrations of organ pipes continued into the current century with Gautier and Tahani publishing a physical model of a simplified cylindrical wind instrument with vibrating walls,[20] Runnemalm *et al.* reporting quantitative measurements of structural vibration modes of an open organ pipe,[21] and Kob confirming once more that vibrations can alter the sound of flue organ pipes.[22,23] And while theoretical investigations did not provide satisfying explanations for vibrational influences, experimental evidence indicating an effect continued to increase.[24]

Eventually, convincing experiments were reported by Moore *et al.* that demonstrated the effect that wall vibrations can have on the sound from a brass wind instrument.[25,26] Moore reported consistent and audible timbre changes on trumpets when the bell was damped using sand bags. These experiments, in contrast to most of the previous ones, used artificial lips to ensure that the driving mechanism was not changed between successive measurements.

Results of similar experiments with French horns mounted in a heavy box have been reported by Kausel *et al.*, demonstrating audible timbre changes of the artificially blown horn while filling the box with dry sand.[27] Later he presented speaker-stimulated transfer function measurements illustrating very similar broadband differences when the horn bell was damped with sand.[28]

Once sufficient experimental evidence of the effects of wall vibrations on the sound of brass wind instruments had been published, several groups began the search for the cause of these effects in earnest. In 2006 Ziegenhals observed that the measured wall vibrations contained all of the frequency components of the radiated sound, with amplitudes approximately comparable to what is found in the sound field.[29] From this, one may surmise that the wall mainly performs forced oscillations while eigenmodes of the structure do not play a dominant role.

By 2008 Nief *et al.* had shown that elliptical modes in the tubing of pipes could affect the sound produced by wind instruments.[30] Their work demonstrated that vibroacoustic coupling between the internal sound field and the walls of the instrument can occur; however, they concluded that these effects probably do not occur in musical instruments with the exception of some thin-walled organ pipes.

A more detailed review up to 2003 can be found in the thesis of Whitehouse, who systematically deals with the effects of different wall materials and thicknesses in simplified wind instruments consisting of a mouthpiece and a cylindrical tube.[31] In these cases the dominant excitation mechanism of structural resonances has been found to be the lip oscillator rather than the resonant air column.

Based on a review of the literature, one can now confidently assert that the wall vibrations of brass wind instruments do indeed affect the radiated sound. Furthermore, this effect can be significant enough to be noticeable to the average listener,[26] and the differences in the power contained in individual overtones can exceed 6 dB in some cases. The issue that we address below concerns the etiology of this effect.

II. EXPERIMENTS AND OBSERVATIONS

As noted above, the results of experiments investigating the effects of damping the wall vibrations of a trumpet using excitation by artificial lips has been reported elsewhere.[26] In those experiments, bags of sand were placed on and around the outside of a trumpet bell to damp the motion and the results showed a significant difference in the sound in the far-field between this case and the case where the bell was left free to vibrate. While it is unnecessary to reprint the results of this work, it is useful to note that the effects described only appear when the vibrations of the bell section are heavily damped. Damping the vibrations of portions of the instrument more than approximately 15 cm from the end of the bell produced no measurable effect on the radiated sound.

While it was necessary to use artificial lips to demonstrate that wall vibrations affect the sound of wind instruments during actual performance, excitation by a more controlled method reveals important information that leads to new insights. To this end, experiments were performed using a 40 W horn driver with a one-inch diameter titanium diaphragm to excite the vibrations of the air column of the trumpet. The driver was attached to the mouthpiece of the trumpet by a 3.8 cm long section of rigid pipe. A condenser microphone connected to a low-noise preamplifier was inserted into the pipe between the horn driver and the mouthpiece to record the amplitude of the input signal. A second microphone was placed at the output plane of the bell and centered on the axis of the bell to record the amplitude of the output signal. The transmission of the instrument was determined by the ratio of the output to the input amplitude over frequencies ranging from 100 Hz to 2 kHz. The pressure produced at the mouthpiece was 87 ± 1 Pa at 1 kHz.

The experiments described here were all performed on a trumpet manufactured by the King Instrument Co. ca. 1970. The trumpet was a *Silver Flair* model and was the same instrument used in the experiments described in Reference 26. The instrument was mounted securely by clamping the valve section to an aluminum platform mounted on an optical table. The mounting structure left the majority of the tubing and the complete bell section free to vibrate. When desired, the mechanical vibrations were damped by placing plastic bags filled with sand around the bell section.

The instrument and mounting hardware were placed behind a 0.75×1.25 m^2 plywood baffle that was covered with anechoic foam. An aperture in the baffle approximately 2 mm larger than the bell diameter allowed the bell to vibrate freely while still ensuring that radiation from behind the baffle was minimized at the position of the microphone. The experimental apparatus was built inside a room that was covered on five sides with anechoic foam. The floor of the room in front of the apparatus was also covered in anechoic foam. This arrangement ensured that physical changes in the surroundings, such as changes in the positions of the sandbags, did not significantly affect the signal recorded by the output microphone. The speaker was driven with a sine wave generated by a computer with a 20 kHz sampling rate and the frequency was swept from 100 Hz to 2 kHz in 1 Hz increments while the signals from the input and output microphones were recorded. The sample time at each frequency was 1 s. After leaving the preamplifiers the signals were amplified and digitally stored for later analysis.

A. Experimental results

Plots of the transfer function of the trumpet are shown in Fig. 1(a). Both the case where the bell is free to vibrate and the case where the bell is heavily damped are shown. There is clearly a difference in the spectra when the structural vibrations are damped, but it is important to note that the change in transmission is neither constant nor consistently increased or attenuated. At low frequencies the amplitude of the signal from the damped bell exceeds that of the freely vibrating bell, but at frequencies above approximately 500 Hz this effect is inverted, and then inverted again at approximately 1.5 kHz. The maximum gain in the amplitude achieved by damping the bell was approximately 1.3 dB, while maximum attenuation was approximately -1.7 dB. The difference between the damped and undamped transmission is shown in Fig. 1(b).

The input impedance of the trumpet was also measured under the same conditions using the Brass Instrument Analysis System (BIAS) manufacture at the Institut für Wiener Klangstil. Changes as large as 6 MΩ (approximately 5%) were observed. The input impedance in both configurations is shown as a function of frequency in Fig. 1(c). The difference between the impedance in the two cases is plotted in Fig. 1(d). It is interesting to note that the change in the impedance with damping changes sign at approximately 900 Hz instead of 500 Hz, and that there is no evidence of a second change at 1.5 kHz as is evident in the transmission function.

Since it is possible that direct radiation from the bell motion may be responsible for the observed effects, the frequencies of the structural resonances of the trumpet bell were measured. To determine the frequencies of the structural resonances, the motion of the bell was observed using time-averaged electronic speckle pattern interferometry. The interferometer had an amplitude resolution of approximately 100 nm and the trumpet was positioned at three angles so that the bell was viewed directly into the bore of the instrument, from the side, and from the back. The speaker was driven by a sinusoidal signal of variable frequency with a resolution of ± 0.05 Hz. Resonances were determined by varying the driving frequency while observing the interferogram in real-time. In this manner, the pattern and frequency of many of the structural modes were measured. As has been noted before, the vibrational patterns of trumpet bells are similar to those of carillons, church bells and hand-bells; therefore, the modal shapes can be uniquely characterized by the number of nodal diameters and nodal circles.[32] The modes that could be excited in this manner are listed with the resonance frequencies in Table I. The amplitude of vibrations of one antinode of each of the observable modes was measured as a function of frequency to determine the bandwidth of each resonance. A fit to the data using a Lorentzian function reveals that the quality factors of the resonances are quite high; these values are also noted in Table I.

Interferograms of the (2, 1) mode taken from the front, side and back of the bell are shown in Fig. 2. The (2, 1) mode is the most easily excitable resonance below 3 kHz, and is of particular interest because it is within the normal

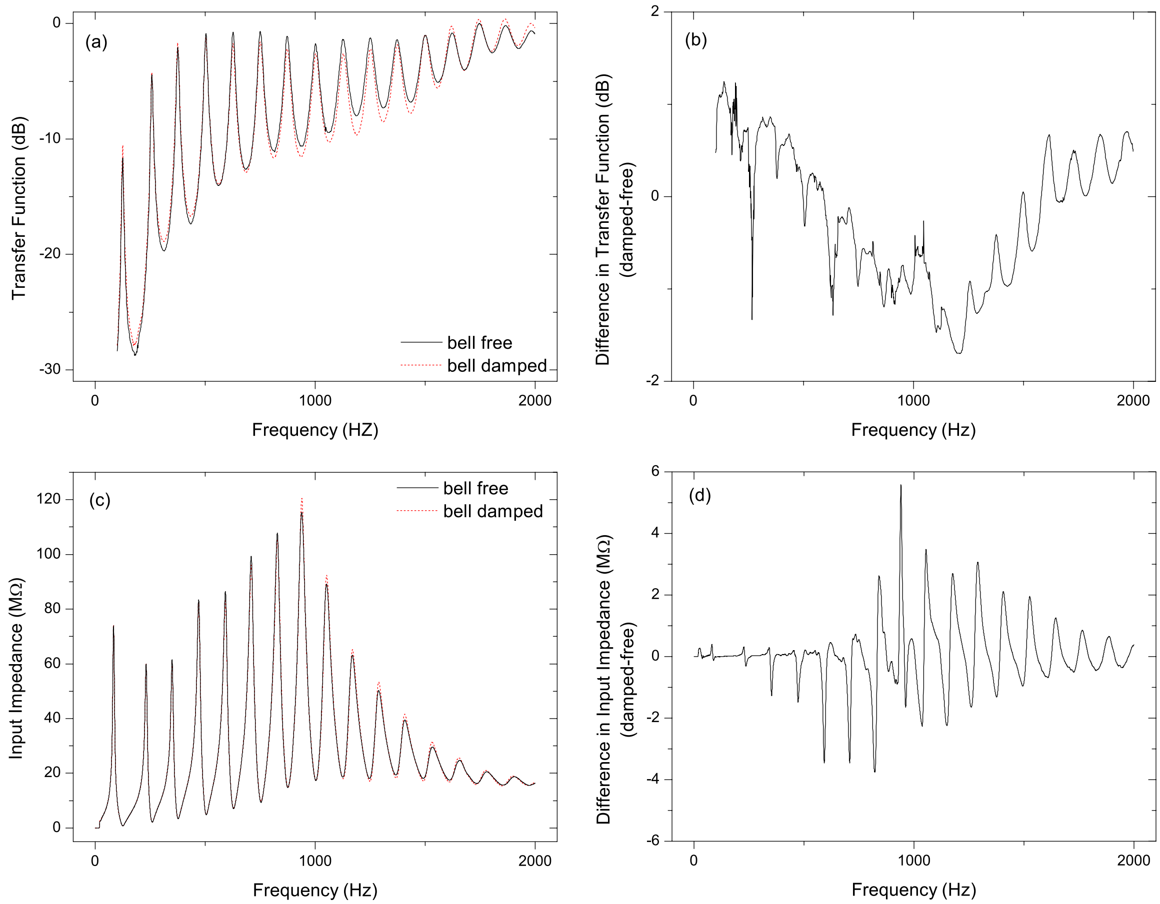

FIG. 1. (Color online) (a) Transmission of the trumpet measured at the output plane of the bell plotted as a function of driving frequency. The two graphs correspond to the bell being free to vibrate (solid line) and the case where the bell is heavily damped with bags of sand (dashed line). (b) The difference in the transmission of the trumpet produced by damping the bell vibrations. (c) Input impedance spectrum of the trumpet measured at the mouthpiece. (d) The difference in the input impedance of the trumpet produced by damping the bell vibrations.

playing range of the instrument and very nearly coincides with a concert B-flat. It is important to note that the deflection shape shown in Fig. 2 is typical, in that all of the deflection shapes at the frequencies of mechanical resonances showed a high degree of symmetry and the degeneracy between orthogonal modes was typically only a few hertz. This indicates that the trumpet bell was highly symmetrical and had only minimal inhomogeneity.

To determine the effects that are attributable to direct radiation from the vibrating metal, a microphone was placed approximately 1 mm from the bell in a position directly over an antinode of the (2, 1) mode. The input was then swept through a frequency range from 464 Hz to 468 Hz in 0.1 Hz intervals while the signal from the microphone was recorded. This frequency range included the frequency of the (2, 1) resonance. The results from measurements taken over two adjacent antinodes at the same axial position are shown in Fig. 3(a). (The difference in the two curves can be attributed to the fact that the two antinodes are out of phase with one another, and therefore the radiated sound can interfere either constructively or destructively with the sound from the air column.) The bell was then damped with sandbags and the measurement repeated. These results are also shown in Fig. 3(a). Results of the same experiment with the microphone placed over a nodal line are shown in Fig. 3(b). The results shown in Figs. 3(a) and 3(b) make it clear that the effects attributable to the direct radiation from the (2, 1) mode are eliminated when the bell vibrations are damped. However, it

TABLE I. Measured frequencies and quality factors of the most easily excited normal modes of the trumpet bell below 3 kHz.

Mode	Frequency (Hz)	Q
(2, 1)	466	737
(2, 2)	1133	242
(3, 2)	1194	685
(3, 2#)	1356	954
(4, 2)	1815	1014
(4, 2#)	2255	817
(5, 3)	2736	1016

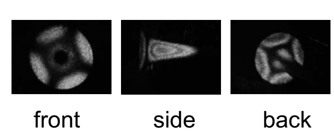

FIG. 2. Time-averaged electronic speckle-pattern interferograms of the lowest and most easily excited mode of the trumpet bell viewed from the front, side and back. The resonance frequency is approximately 466 Hz.

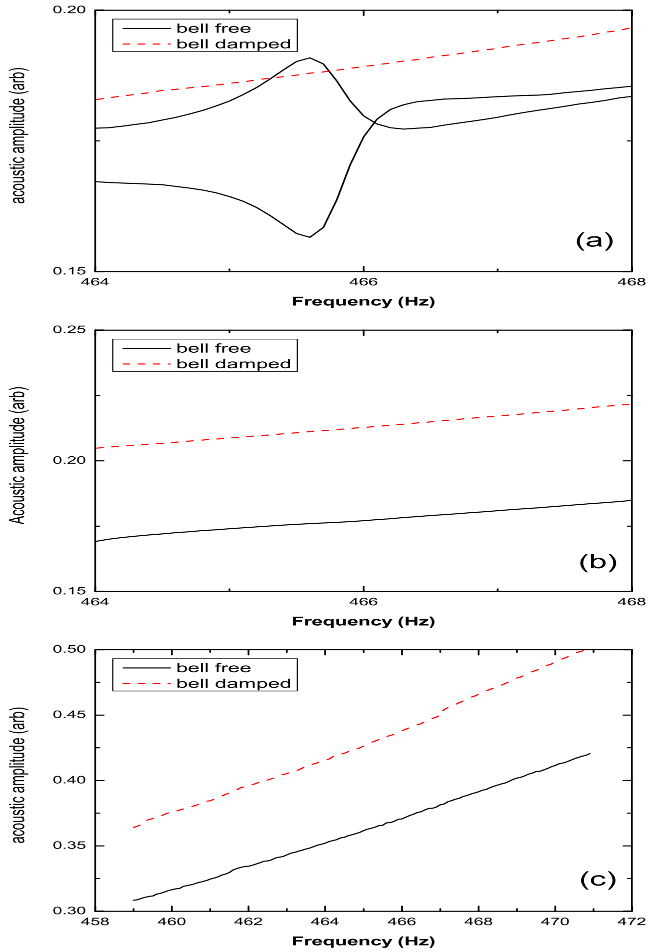

FIG. 3. (Color online) (a) Amplitude of the acoustic field directly over two adjacent antinodes of the (2, 1) mode on the bell (solid), plotted with the amplitude over one of them when the bell vibrations are heavily damped (dashed). The frequency range includes the resonance frequency of the (2, 1) mode. (b) Amplitude of the acoustic field directly over a node of the (2, 1) mode on the bell (solid), plotted with the amplitude at the same point when the bell vibrations are heavily damped (dashed). The frequency range includes the resonance frequency of the (2, 1) mode. (c) Amplitude of the acoustic field 1 m from the bell over a frequency range that includes the (2, 1) resonance for the case of the bell left free to vibrate (solid) and heavily damped (dashed).

is also clear that there is a broad-band phenomenon not related to the specific resonance that enhances the sound when the bell is damped.

Finally, the trumpet was replaced with a piece of straight brass tubing. A standard trumpet mouthpiece receiver was soldered to a leadpipe, which was then connected to 76 cm of 12.5 mm diameter tubing. The total length of the receiver, leadpipe and tubing was 1.09 m. The driving mechanism was the same as described above, with the horn driver attached to the mouthpiece and driven by a sinusoidal input. The tubing was suspended by two wires so that it was free to vibrate. As with the experiments described above, the sound from the open end of the tubing was recorded as the input frequency was swept from 100 Hz to 2 kHz in increments of 1 Hz, then the tubing was heavily damped with bags of sand along its entire length and the experiment was repeated. This experiment was performed several times, and in each case there was no significant difference between the recorded sound when the tubing was free to vibrate and when the vibrations were damped.

B. Analysis of experimental results

In Reference 26 it was hypothesized that the effects attributable to the vibrations of the bell may be explained by mechanical feedback to the lips. However, the results presented here indicate that this is not the case. Damping the bell vibrations results in large variations in the transmission function when the input stimulus is produced by electronic means, therefore it is unlikely that mechanical feedback plays a significant role in creating the observed effects. Furthermore, variations in the transmission function and input impedance are absent when the vibrations are damped at places other than the bell section, and are also absent when the trumpet is replaced by straight cylindrical pipe. Therefore, it is likely that the observed effects are due to processes that are related directly to the motion of the bell section and will not be observed in non-flaring pipes.

While it is likely that the observed changes in the sound are due to vibrations of the bell section, it is unlikely that direct radiation from the bell can account for a significant portion of these observed effects. Although the mechanical resonances are easily excited, the range of frequencies that will excite them is extremely narrow. Since the effects attributable to bell vibrations have been shown to occur over a broad frequency range, it is difficult to believe that the origin of these effects can be traced to a series of high-Q resonances that are widely separated in frequency. Furthermore, the symmetry of the modes will lead to significant acoustic short-circuiting, resulting in an effect that will be difficult to detect in the far-field. This is demonstrated in Fig. 3(c), which contains a plot of the amplitude of the sound field directly in front of the bell approximately 1 m away as a function of frequency over a narrow range that includes the (2, 1) resonance. The resolution is 0.1 Hz. The data in Fig. 3(c) indicate that there are no noticeable effects attributable to this easily excited resonance in the far field, even though the results shown in Fig. 3(a) clearly indicate significant bell vibration in that frequency range. The measurements of the sound field over a node in the modal structure shown in Fig. 3(b) also indicate that damping the bell vibrations results in a significant change in the sound that has little to do with direct radiation from the vibrating metal.

Although it appears that direct radiation cannot account for the acoustic effects attributable to bell motion, and the narrow structural resonances are likewise unlikely to explain them, the data indicate that there is a strong correlation between changes in the sound field and changes in the motion of the bell. All of the data presented above clearly indicate that damping the vibrations of the bell significantly affects the radiated sound in a manner not associated with the motion of the lips. Therefore, there must be some interaction between the vibrating metal and the sound transmitted through the air column that can account for the observed effects. We investigate one possibility below.

III. MODELING WALL VIBRATIONS

The experimental observations presented in the previous section indicate that the mechanism through which the sound field is influenced by the vibrating wall is not strongly dependent on the detectable structural resonances. This effect, which is not frequency-independent, but occurs over a broad range of frequencies, is evidently larger than the effects attributable to the known structural resonances and is significant enough to be consistently measured. Indeed, the effect can be significant enough to be audible to an untrained person and it usually affects the whole playing range, not just a few discrete frequencies or very narrow frequency bands.[26,27]

Elastic strain of the metal wall, which is proportional to the oscillating sound pressure inside the instrument, could provide an explanation for the observed shifts in input impedance and changes in the transfer functions. This kind of forced strain oscillation is usually ignored in the analysis of wind instruments because it is difficult to imagine that those very small pressure forces which are present inside a brass wind instrument or an organ pipe could be large enough to cause any significant modulation of the effective cross-sectional area of the bore. However, a first order numerical analysis of such interactions reveals differences in the impedance and transfer function that are of the right order of magnitude and qualitatively similar to the experimental observations.

In what follows we estimate the influence on the sound field that a reasonably sized oscillating change in cross-sectional area may produce on the impedance and pressure transfer function. We refer to these axially symmetric changes in the bore diameter as *breathing modes*. The analysis is accomplished by including the effect of an oscillating volume of the cylindrical or conical bore slices in the widely used transmission line model of acoustical ducts.

A. Static case

We begin by calculating the static (purely proportional) case, where the change in diameter of a cylindrical pipe is directly proportional to the applied pressure. A general approach can be found in the literature;[33] however, for the case considered here a more simple derivation is adequate.

In the standard transmission-line model, the instrument is divided into a series of short segments as shown in Fig. 4. Since the wall thickness is small compared with the tube radius, there will not be significant stress in the radial direction. Stress in the axial direction associated with Poisson's ratio and by forces related to pressure gradients are also neglected. Since the pressure distribution along the instrument axis is smooth and never discontinuous, these assumptions also apply to the dynamic case considered below.

The balance of forces inside any short segment of length Δx of a pressurized pipe with internal pressure p, radius r and wall thickness t, requires the pressure induced force $F_\sigma = p(2r\Delta x)$ on any cross-section to be compensated by an equal total force in the walls. The hoop stress σ due to the internal air pressure p can therefore be calculated by

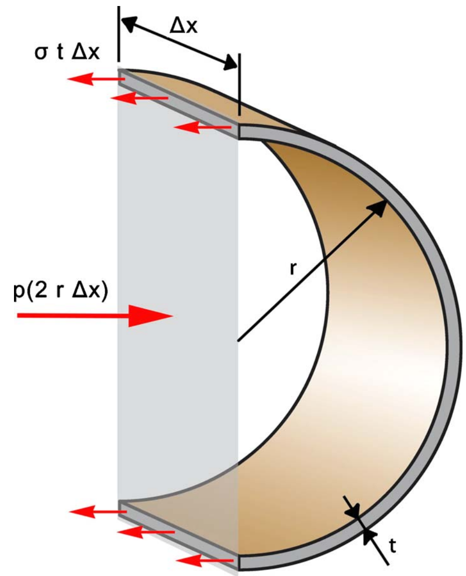

FIG. 4. (Color online) Diagram of the wall forces caused by internal pressure.

$$\sigma = \frac{pr}{t}. \tag{2}$$

From the axially symmetric hoop stress σ the relative change of circumference ε is given by

$$\varepsilon = \frac{\sigma}{E}, \tag{3}$$

where E represents the Young's modulus of the material, which is approximately 100–125 GPa for brass. The relative change in the circumference leads to a larger cross sectional area of the pipe when an internal positive pressure is present. The change in pipe radius Δr over the causative pressure p is then given by

$$\frac{\Delta r}{p} = \frac{r^2}{Et}. \tag{4}$$

A quasi-static view, considering frequencies small enough to neglect mass inertia of the wall as well as its internal friction against strain, allows one to define the change in the pipe radius at the maximum positive instantaneous pressure $\hat{p}$ as the amplitude of the oscillations in the wall displacement $\hat{s}$,

$$\hat{s} = \hat{p}\frac{r^2}{Et}. \tag{5}$$

B. Flaring sections

If the tube is not completely cylindrical but instead flares, as in the bell region of most brass instruments, the situation must be reconsidered. The flare angle φ that the wall makes with the axis of the instrument ranges between zero (cylindrical parts) and approximately 70° in trumpets, and even more in certain other instruments.

Figure 5 shows how the air pressure acting perpendicular to the wall will displace the wall. Since we are interested

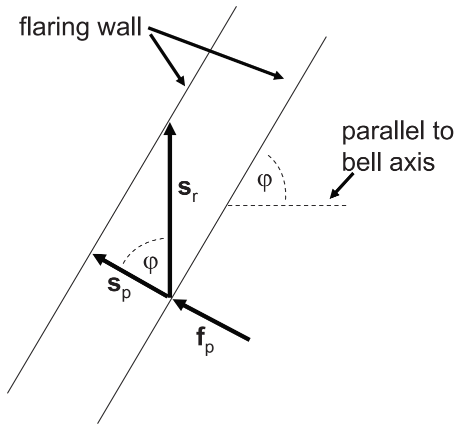

FIG. 5. Radial wall displacement s_r due to perpendicular displacement s_p of flaring section of the wall caused by internal force f_p. The solid diagonal lines represent the wall surface in its original and displaced position.

in the cross-sectional area of a conical slice, which is described in a coordinate system fixed to the air column, the displacement normal to the wall s_p caused by the pressure force $f_p \approx p2r\pi(\Delta x/\cos(\varphi))$ is translated into a radial shift $s_r = s_p/\cos(\varphi)$.

The displacement s_p will also be larger than that found in the purely cylindrical case because the direction of the displacement is no longer radial, so less circumferential strain is required for the same amount of displacement. This can be taken into account by another factor of $\cos(\varphi)$, which reduces the effective Young's modulus in that case accordingly. This simplification requires that Δx be small enough that the difference between the right and left radius of the conical segment is much smaller than the radius itself.

It makes sense to reconsider this reduction when the flare angle comes close to 90°. Although it is true that no circumferential strain is required to displace the edge of the bell where the brass is perpendicular to the axis, bending stiffness will have to be overcome to move the brass. In Section III C 2 it will be shown that the effective elasticity constant in this case will be at least one order of magnitude smaller than in the cylindrical case [Eq. (11)]. So the reduction of the effective elasticity constant with $\cos(\varphi)$ should be constrained in order that it never go below approximately $E/10$ for bells with flare angles above about 84°.

The actual bending stiffness is a function of the shape as well as the material, and therefore must be determined experimentally for any specific instrument bell. Since the trumpet under consideration does not flare so steeply that $\cos(\varphi)$ reduces E by more than an order of magnitude, this special case is neglected and Eq. (5) can be rewritten as

$$\hat{s} \approx \hat{p}\frac{r^2}{Et\cos(\varphi)^3}. \tag{6}$$

From the amplitude of the wall displacement we can derive the wall velocity $\hat{v} = \omega\hat{s}$ and the parasitic flow $\hat{u}_L = \hat{v}2r\pi\Delta x$, which is lost into the vibrating wall.

C. Non circular cross section

Brass wind instruments are usually treated as having a perfect axial symmetry with truly circular cross-sections; however, this is normally not the case. Even disregarding the fact that wind instrument tubes are very often toroidally bent, they are still subject to manufacturing tolerances and deformations even if those are too small to be visible to the naked eye.

Given the inherent absence of perfect circular symmetry, one can imagine that there are at least two different kinds of deformations which might produce effects that are not negligible. First, the motion of elliptical modes of the cylindrical walls may change the cross sectional area of the bore. Similarly, the internal pressure may expand bore deformations, bringing the bore closer to being perfectly circular. In either case, the change in cross sectional area will result in a change in the transfer function. The order of magnitude of both effects is investigated below to determine whether such mechanisms can become acoustically relevant, and whether they might be responsible for at least a portion of the effects that have consistently been observed in the experiments.

1. Elliptic oscillation modes

The acoustical effect of an elliptical oscillation of the structure has already been studied by others.[30] In spite of the fact that elliptic modes are easy to excite and usually exhibit large wall displacement amplitudes, their presence probably cannot explain all of the effects observed in the experiments described above.

The most obvious reason that elliptical motion of the walls cannot account for all of the observed effects is that this type of mode always occurs over a narrow frequency range. As noted in Section II A, interferometry and acoustic data indicate that the observed elliptical modes in most brass wind instruments have a high Q. These motions can radiate sound in the near field, which explains the data shown in Fig. 3(a); however, all of the data indicate that the more important effects occur over a broad frequency range. Therefore, to be responsible for the observed effects the mechanism that is modulating the cross-sectional area of the bore must occur over a wide range of frequencies.

It is possible that elliptical modes can modulate the cross-sectional bore area over a wide frequency range below their eigenfrequency in cases where an existing elliptic deformation is periodically made circular by the internal sound pressure. Using the second order approximation of the arc length of an ellipse, which is very accurate for nearly circular shapes, we can express the cross-sectional area $A_{ellipse}$ over the length of the major axis at constant perimeter as a solution to the non linear system of equations

$$I: \frac{(9a^2+14ab+9b^2)^2\pi}{64(a+b)^3} = 2\pi,$$

$$II: \quad A_{ellipse} = ab\pi, \tag{7}$$

where a and b are the lengths of the semi-major and semi-minor axes. The relative change in area is shown in Fig. 6 as a function of the ellipticity, where it can be seen that the dependence is extremely weak. For example, a 1% deviation in radius from the perfect circle causes an area difference of only approximately 0.015%. No considerable bore area

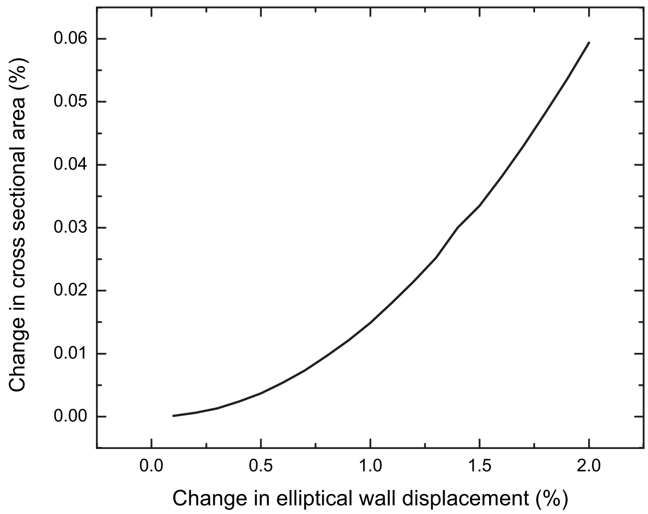

FIG. 6. Relative change in the cross sectional area caused by elliptical wall displacement.

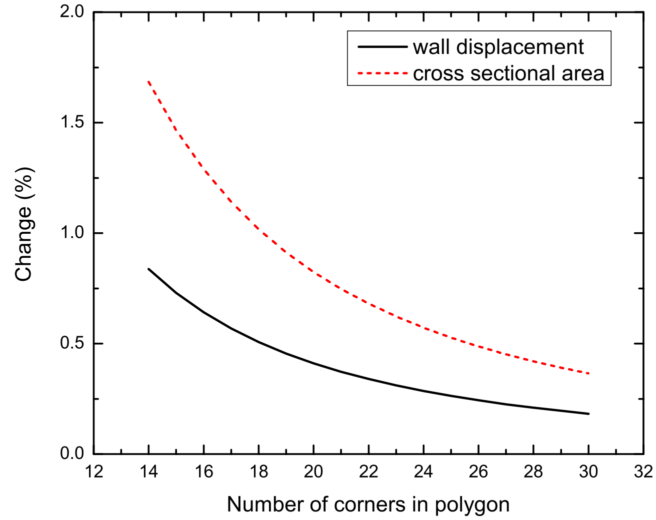

FIG. 7. (Color online) Relative change in cross-sectional area (dashed) and relative wall displacement (solid) for the case of straightening a polygon toward a circle, plotted as a function of the number of corners.

modulation can be expected from this kind of forced oscillation.

2. Irregular deformations

Manually made bells are especially susceptible to random deviations from the perfectly axial symmetrical shape. To estimate the effect of small and usually invisible irregularities in the shape, regular polygon shaped cross-sections can be analyzed and compared with the effect of an elliptical deformation. Area $A_{polygon}$, and outer and inner radii r_{outer} and r_{inner}, of a regular polygon with n corners are given by

$$A_{polygon} = \frac{n}{4}\cot\left(\frac{\pi}{n}\right),$$

$$r_{outer} = \frac{\csc\left(\frac{\pi}{n}\right)}{2},$$

$$r_{inner} = \frac{\cot\left(\frac{\pi}{n}\right)}{2}. \tag{8}$$

In a regular polygon with 18 corners the outer radius differs from the equivalent circle radius by not more than one-half of one percent. In the acoustically most sensitive region of a trumpet bell this corresponds to a deformation depth of about 0.125 mm which would usually be undetectable.

Figure 7 shows the much stronger relationship between bore area and wall displacement than occurs with elliptical deformation. In Fig. 7 the change in the cross sectional area is plotted as a function of the number of corners of a regular polygon. Regardless of the number of corners, it can be stated that a wall displacement of 1% causes a bore area difference of about 2%, which is comparable to that of a strain oscillation.

Oscillations of these shapes become more interesting than simple strain oscillations when interactions between wall vibrations and the air column are considered. Due to the fact that the metal does not need to be stretched along the entire circumference, these oscillations will be more easily excited than simple strain oscillations. For the case of a pipe deformed into the shape of a regular polygon, the pressure forces must act only against the bending stiffness of a thin metal sheet, which decreases with the third power of the wall thickness.

The center displacement s of a flat spring of length L, thickness t, width b and Young's modulus E caused by a central force F_r, as shown in Fig. 8, is given by

$$s = \frac{F_r L^3}{4bEt^3}. \tag{9}$$

The central force F_r originates from circumferential forces F_t and is given by $F_r{=}2F_t s_0/L$. Solving for the circumferential force F_t we obtain

$$F_t = \frac{2Ebst^3}{L^2 s_0}. \tag{10}$$

Introducing the hoop stress $\sigma{=}F_t/bt$ and the relative length change $\varepsilon{=}\Delta r/r{=}s/r$, we can define an equivalent elastic constant $E^*{=}\sigma/\varepsilon$ which results in

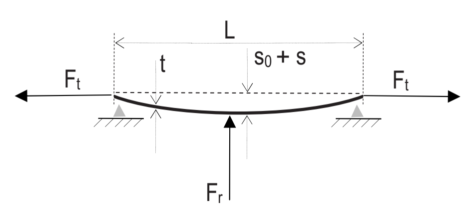

FIG. 8. Diagram of forces on a flat spring.

$$E^* = E\frac{2rt^2}{L^2 s_0}. \quad (11)$$

Assuming a region inside a trumpet bell at a bore radius of 5 cm with a brass thickness of 0.3 mm, the equivalent elastic constant E^* becomes an order of magnitude smaller than the Young's modulus of the brass when the deformations are comparable to an 18 sided polygon (a typical bore inaccuracy of 0.5%).

D. Dynamic case

It is clear from the data shown in Fig. 1(a) that there are critical frequencies near 500 Hz and 1500 Hz where the shift in the pressure transfer function caused by damping the wall vibrations changes sign. Below about 500 Hz vibrations of the bell seem to have an absorbing effect, while above it there is an obvious increase in the radiated power. Above about 1500 Hz the effect is reversed again.

If mass inertia of the wall and internal losses are added to the model then the presence of a critical frequency is expected. At frequencies above a resonance frequency wall displacement will not be in phase with the causative sound pressure; however, at the critical frequency there will be a phase transition from 0 to $-\pi$, which will invert any effect that is dependent upon the relative phase between the motion and the oscillating pressure. Although a single mass model cannot explain the occurrence of more than one critical frequency, it will help to understand what might actually happen to the wall.

It should be noted here that a single mass oscillator with a high-Q resonance would have the strongest effect in a narrow region around the critical frequency, but this is not the effect evident in Fig. 1(a). On the other hand, a low-Q resonance with strong damping would describe the effect, but does not seem to be physical because the elastic strain far below the yield threshold is known to be an almost lossless process. Therefore, low order modes usually exhibit Q-factors well above 200 as shown in Table I. This was also shown by Yoshikawa,[34] although he did not investigate this phenomenon in brass tubes. This difficulty will be addressed below, after the single mass model has been developed.

Any damped mechanical system of first order which is driven by a sinusoidally oscillating external force with amplitude $\hat{F}$ and frequency ω can be described by

$$ks(t) + \gamma s'(t) + ms''(t) = \hat{F}e^{\mathrm{i}\omega t}, \quad (12)$$

where k is the effective spring constant, γ is a damping coefficient and $s(t)$ is the displacement of the effective mass m. The velocity and acceleration are denoted as usual by $s' = ds/dt$ and $s'' = d^2s/dt^2$ respectively.

The spring constant k, effective mass m, and damping coefficient γ are rather abstract quantities in the situation under consideration here; therefore, it is useful to describe the motion in terms of the resonance frequency $\omega_0 = 2\pi f_0$, the quality factor Q, and the quasi-static displacement amplitude $\hat{s}$ at very low frequencies. The first two of these quantities can be measured, while $\hat{s}$ can be calculated using Eq. (5) or Eq. (6).

With these quantities we can formulate Eq. (12) as

$$\frac{s''}{\omega_0{}^2} + \frac{s'}{Q\omega_0} + s = \hat{s}e^{\mathrm{i}\omega t}, \quad (13)$$

thus defining the correspondences

$$m = \frac{k}{\omega_0{}^2}, \quad (14)$$

$$\gamma = \frac{k}{Q\omega_0}, \quad (15)$$

and

$$\hat{F} = k\hat{s}. \quad (16)$$

1. Amplitude and phase of wall vibrations

The solution for the displacement s is again a harmonically oscillating function $s(t) = \hat{A}(\omega)e^{\mathrm{i}\omega t}$ with an amplitude $\hat{A}(\omega)$, which can be obtained by substituting $s(t)$ and its derivatives $s'(t)$ and $s''(t)$ into Eq. (13). Doing so results in

$$\hat{A}(\omega) = \hat{s}\frac{\omega_0{}^2}{\omega_0{}^2 + \mathrm{i}\omega\frac{\omega_0}{Q} - \omega^2}. \quad (17)$$

The magnitude of the displacement $|\hat{A}(\omega)|$ is then given by

$$|\hat{A}(\omega)| = \hat{s}\sqrt{\frac{\omega_0{}^4}{\omega^4 + (Q^{-2} - 2)\omega^2\omega_0{}^2 + \omega_0{}^4}}, \quad (18)$$

with the phase being

$$\arg(\hat{A}(\omega)) = \arctan\frac{-\omega\omega_0}{Q(\omega_0^2 - \omega^2)}. \quad (19)$$

2. Critical frequency of strain oscillations

If we concentrate the whole mass of one hoop section and introduce an effective spring constant k for the radial displacement s, we can calculate the critical frequency according to $\omega_0 = \sqrt{k/m}$. Since we expect quality factors to be well above unity, the reduction in the resonance frequency $\omega_R = \omega_0\sqrt{1 - 1/2Q^2}$ caused by damping can be ignored.

For a conical hoop segment of infinitesimal width Δx the total mass is given by

$$m = 2r\pi\frac{\Delta x}{\cos(\varphi)}t\rho, \quad (20)$$

with ρ being the mass density of the wall material. Using the definition of the spring constant, $k = F/s$, and substituting the total radial force $F = p2r\pi\Delta x$, and s given by Eq. (6) results in

$$k = \frac{2\pi\Delta x Et\cos(\varphi)^3}{r}. \quad (21)$$

This leads to an expression for the critical frequency,

$$\omega_0 = \sqrt{\frac{E\cos(\varphi)^4}{r^2\rho}} = \frac{\cos(\varphi)^2}{r}\sqrt{\frac{E}{\rho}}. \qquad (22)$$

Note that the critical frequency does not depend on the wall thickness t, but it depends strongly on the flare angle φ and the radius r.

The radius and flare angle dependencies indicate that there is not one single critical frequency, but a spatially distributed range of critical frequencies. That is, there are regional resonances that are excited at different positions as the driving frequency of the air column changes. Thus the wide range of bore radii and flare angles in the bell of typical brass wind instruments causes strain oscillation resonances over a wide range of frequencies, but local to different parts of the bell.

Because strain resonances are assumed to be high-Q, this position-dependent resonance would lead to strong gradients of the displacement amplitude along the axis of the instrument, which clearly violates the initial assumption of negligible inter-segment forces. The effect of these forces can be assumed to reduce the amplitude at the position of maximal motion and increase the amplitudes in the surrounding region. To approximate this expected kind of spatial filtering without rigorously changing the simplified structural mechanics, a simple smoothing algorithm has been applied to the predicted amplitude distribution.

The rectangular spatial smoothing kernel used for calculating a running average of displacement amplitudes had a width of 1 cm with a bore resolution of 0.5 mm, and therefore consisted of 20 values. This kernel was chosen with the goal of adjusting the predicted wall vibration effects in such a way as to match the observations as well as possible without introducing unreasonable assumptions. Using a filtering technique such as this to compensate for the obvious violation of initial assumptions, which justified the use of independent hoop sections, is not very elegant; however, the purpose of this work is to propose a theory that will account for the experimentally observed effects. While a rigorous treatment of the structural mechanics of the actual bell geometry is unnecessary to accomplish the stated goals of this work, it will be subject of future research.

3. Frictional losses in the wall

The equation of motion [Eq. (12)] contains a damping term where a frictional force $\gamma s'(t)$ causes energy dissipation during each vibration cycle. If $s(t)$ is assumed to be harmonic, $s(t)=\hat{A}\sin(\omega t)$, then $s'(t)=\hat{A}\omega\cos(\omega t)$ and $ds=\hat{A}\omega\cos(\omega t)dt$. The energy loss per cycle then becomes

$$W_L = \int_S \gamma s'(t)ds = 4\int_0^{\pi/(2\omega)} \gamma s'(t)^2 dt = \gamma\pi\omega\hat{A}^2. \qquad (23)$$

Multiplying the energy loss per cycle by the frequency $f=\omega/2\pi$ and substituting γ from Eq. (15), k from Eq. (16) and $\hat{F}=\hat{p}2r\pi\Delta x$ we obtain the total power loss P_L in a thin slice of length Δx due to friction in the vibrating wall as

$$P_L = \frac{\omega}{2\pi}W_L = \hat{p}\frac{r\pi\Delta x}{\hat{s}Q\omega_0}\omega^2\hat{A}^2. \qquad (24)$$

When pressure p and flow u are propagated, power dissipation due to friction can be taken into account by modifying the exit pressure in a way to reduce the active power output of the air column $P_{act}=\hat{p}\hat{u}\cos(\varphi_{p,u})$. The reduction P_L corresponds to the power dissipated in the vibrating wall of that element.

No assumptions are made here about the physical interpretation of the overall damping constant γ. It may originate from inner friction against strain, but may also be due to the air load of the outer wall surface. However, numerical analysis has shown that the contribution due to this damping coefficient γ is negligible compared to the contribution discussed in the next section.

4. Thermodynamic pressure modulation due to volume oscillations

The ideal gas equation $p(t)V(t)=RTn(t)$ relates the number of moles of gas $n(t)$, the volume $V(t)$ and the pressure $p(t)$ at constant temperature T at any time t (R being the universal gas constant). The time varying quantities $p(t)$, $V(t)$ and $n(t)$ are usually derived from constant equilibrium conditions p_0, V_0 and n_0 and small harmonically oscillating magnitudes $\hat{p}e^{\mathrm{i}\omega t}$, $\hat{V}e^{\mathrm{i}\omega t}$ and $\hat{n}e^{\mathrm{i}\omega t}$. The value of $\hat{n}$ can be calculated from the ideal gas equation at constant volume V_0 and constant temperature T by $\hat{n}=V_0\hat{p}/RT$. Isothermal conditions are assumed because the small volume changes being considered occur very close to the metal wall.

Superimposing such volume oscillations with an amplitude $\hat{V}$, which has a phase shift of ϑ with respect to the pressure in the air column, the gas equation becomes

$$(p_0 + p_+(t))(V_0 + \hat{V}e^{\mathrm{i}\omega t}e^{\mathrm{i}\vartheta}) = RT\left(n_0 + \frac{V_0\hat{p}}{RT}e^{\mathrm{i}\omega t}\right). \qquad (25)$$

Neglecting second order terms and solving for the effective time varying pressure $p_+(t)$ yields

$$p_+(t) = \hat{p}e^{\mathrm{i}\omega t} - \frac{p_0}{V_0}\hat{V}e^{\mathrm{i}\vartheta}e^{\mathrm{i}\omega t}. \qquad (26)$$

As expected, the effective pressure is composed of the oscillating pressure, which originally modulated $n(t)$, and an oscillating but phase-shifted additional pressure, which is due to the oscillating volume.

This extra pressure amplitude $\widehat{p_V}$, which is caused by the wall vibrations, is given by

$$\widehat{p_V} = \frac{p_0}{V_0}\hat{V}e^{\mathrm{i}\vartheta} = \frac{p_0}{r^2\pi\Delta x}(2r\pi\Delta x\hat{s})e^{\mathrm{i}\vartheta} = \frac{2p_0}{r}\hat{s}e^{\mathrm{i}\vartheta}. \qquad (27)$$

E. Impedance and pressure transfer function

In one-dimensional transmission line theory complex wave quantities p and u are propagated through sections of arbitrary acoustical ducts according to

$$p_1 = ap_2 + bu_2,$$

$$u_1 = cp_2 + du_2, \quad (28)$$

a, b, c, d being complex frequency-dependent elements of the propagation matrix A which for lossless cylindrical elements is given by[35]

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} \cos(kL) & \mathrm{i}R_0 \sin(kL) \\ \dfrac{\mathrm{i}}{R_0}\sin(kL) & \cos(kL) \end{pmatrix}, \quad (29)$$

with L being the length of the cylindrical section, the wave number $k=\omega/v$ and the characteristic impedance $R_0=\rho_a v/S$. As usual, v is the speed of sound, ρ_a is the density of air and ω is the angular frequency. The proportionality of c and the inverse proportionality of b to the cross-sectional area S also hold in the lossy as well as in the conical case.

If the resulting pressure p_1 is decreased by an amount $p_L=k_p p_1$ due to dissipation in the vibrating wall, and by the (dominating) amount $p_V=k_V p_1$ caused by the oscillating volume, then a corrected left side pressure p_1^* is obtained. This correction results from a corrected matrix element b^* because the matrix element b is proportional to the characteristic impedance R_0 and therefore inversely proportional to the effective cross-sectional area. The matrix element c is also proportional to the effective cross-sectional area and needs to be adjusted accordingly. These considerations can be formulated according to

$$p_1^* = p_1(1 - k_p - k_V),$$

$$p_1^* = ap_2 + b^* u_2,$$

$$\frac{c^*}{c} = \frac{b}{b^*}. \quad (30)$$

Note that all wave quantities p_i and u_i as well as the coefficients a,b,c,d,k_p and k_V are complex and therefore represent an amplitude or scale factor as well as a relative phase or phase shift. Using Eq. (28) and the fact that $Z_2 = p_2/u_2$, we obtain modified matrix elements

$$b^* = b(1 - k_p - k_V) - aZ_2(k_p + k_V),$$

$$c^* = c\frac{b}{b^*}, \quad (31)$$

which now take wall vibration effects into account. The acoustic impedance therefore propagates through ducts with vibrating walls according to

$$Z_1 = \frac{b^* + aZ_2}{d + c^* Z_2}, \quad (32)$$

which allows one to calculate the effective propagation coefficients b^* and c^* during accumulation of all propagation matrices when the accumulation process is started at the known radiation impedance at the open mouth of the bell. These modifications due to wall vibration effects can be interpreted as the static cross-sectional area of an element being slightly increased when a non-rigid wall yields to the air column pressure.

F. Theoretical results

A one-dimensional transmission line simulation using lossy cylindrical and conical elements as proposed by Mapes-Riordan,[35] implemented in the Brass Instrument Analysis System (BIAS), was used to calculate input impedance and mouthpiece-to-bell pressure transfer spectra of the *Silver Flair* trumpet that was used in the experiments reported above. Beginning with the measured bore profile of the bell section, the remaining bore profile was determined by an acoustical reconstruction technique previously described by Kausel.[36] The mismatch between the experimentally determined input impedance of the trumpet and the corresponding theoretical curve of the reconstructed bore profile averaged approximately 10%. The resonance frequencies of all playable notes were accurate to within approximately 15 cents and the difference in the magnitudes of the impedance maxima were less than 1 dB. The theoretical impedance was calculated and then optimized using the first three modes of a modal decomposition described by Kemp.[37] The model was composed from 2728 slices of 0.5 mm length.

In calculating the input impedance and transfer function, all of the matrix corrections outlined above except for the parasitic flow into the vibrating wall discussed in Section III B were taken into account. The loss of energy due to the inner friction of the brass associated with the strain oscillations derived in Section III D 3 and the thermodynamic pressure modulation due to the oscillating volume of the duct elements as outlined in Section III D 4 were both taken into account, with the latter being the dominant effect. However, the Young's modulus of brass ($\sim 10^{11}$ N/m^2) has not been lowered as indicated by Eq. (11). That is, we have not taken into account the possibility that the wall of the instrument may not be perfectly cylindrical and therefore easier to deform. Instead, a perfectly cylindrical wall with 0.3 mm thickness with no deformations over the circumference has been assumed.

Distributed strain resonance frequencies according to Eq. (22) have been used when applying Eqs. (18) and (19). The quality factor of these local resonances was set to $Q=40$ and the simple spatial smoothing filter was applied as described above. The numerical stability of the matrix multiplications was verified by repeating the same simulation with double precision arithmetic and extended precision. The numerical differences between both results were several orders of magnitude smaller than the effects attributable to wall motion.

It is tempting to compare the results of modeling a vibrating wall structure with that of the same structure with rigid walls. However, a comparison with the case of completely rigid walls does not reflect the experimental conditions of damping vibrations with bags filled with sand. Instead of assuming rigid walls, the addition of the sand was modeled by an increase in the mass of the wall and the introduction of heavy damping. The "damped" case was calculated with $Q=5$ and a material density increased by a factor of 100, compared to the "free" case. Using these "damped" parameters leads to a reduction of the displacement ampli-

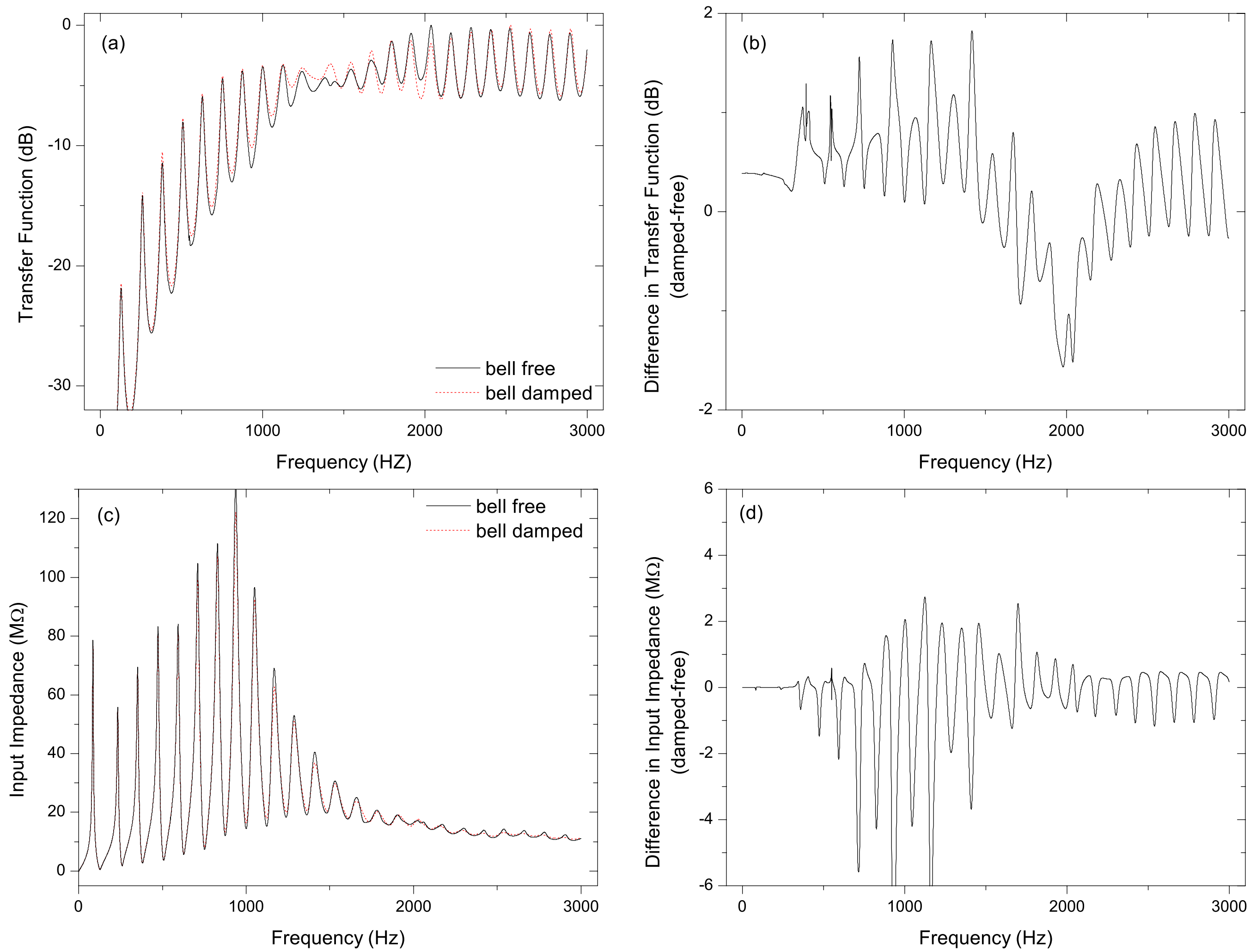

FIG. 9. (Color online) (a) Results of the model for the transfer function of the trumpet with walls free to vibrate (solid line) and with the walls heavily damped (dashed line). (b) Difference in the theoretical transfer functions. (c) Results of model for the input impedance at the mouthpiece of the trumpet with walls free to vibrate (solid line) and with the vibrations heavily damped (dashed line). (d) Difference in the theoretical input impedance.

tudes by approximately one order of magnitude, which is consistent with the measured change reported by Moore *et al.*[26]

Figure 9(a) shows the predicted pressure transfer spectrum for the case with the vibrations damped and with no damping. This can be compared with the experimental data shown in Fig. 1(a). The difference between the two cases is shown in Fig. 9(b), and can be compared to Fig. 1(b). The magnitudes of the differences between the two cases predicted by the theory are quite similar to those observed experimentally, as is the qualitative shape of the graph. The most noticeable difference between the theory and experiment is that the resonance is predicted to occur at a higher frequency than it actually does. The inversion of the effect attributable to damping that occurs at approximately 500 Hz is predicted to occur at approximately 1.4 kHz. The second inversion, which is experimentally measured to occur at approximately 1.5 kHz is predicted to occur at approximately 2.8 kHz.

Similar results are found when the theoretical input impedance spectrum is compared to the experimentally measured impedance. The impedance spectrum predicted by the model is shown in Fig. 9(c) is in excellent agreement with the experimental plot in Fig. 1(c). Also, the difference in input impedance between the damped and freely vibrating case, which is shown in Fig. 9(d), is quite similar to Fig. 1(d). As is the case with the transfer function, the predicted impedance difference between the damped and free case is similar in magnitude, and a graph of the difference is qualitatively similar in shape to the experimentally derived values. The reversal of the sign of the effects attributable to damping, however, is again predicted to occur at a higher frequency than what is measured. Experimentally, the magnitude of the impedance maxima are decreased by damping below approximately 1 kHz and increased above that frequency. Theoretically this change is predicted to occur at approximately 1.5 kHz.

Figure 10 shows the calculated spatial distribution of vibration amplitudes when a 250 Pa sinusoidal pressure source is present in the mouthpiece. This corresponds to a sound level of 142 dB, which can easily be achieved in the mouthpiece of a trumpet. It can be seen that the region with the largest displacement is very close to the rim of the bell, and that the displacement can be more than three orders of

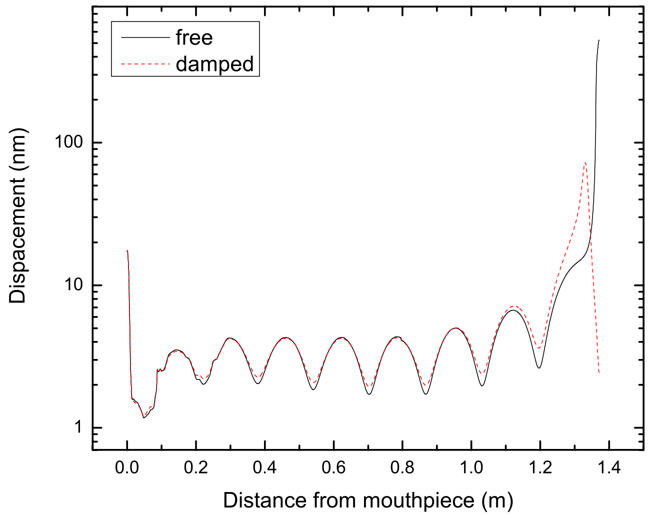

FIG. 10. (Color online) Theoretical wall displacement as a function of position in the trumpet due to strain oscillations caused by a 250 Pa, 1043 Hz sinusoidal stimulus in the mouthpiece. The solid line represents the case of no damping. The dashed line represents the case of heavily damped walls.

magnitude larger than the displacement in the cylindrical parts. This probably explains why the acoustical effects attributable to wall vibrations are difficult to observe in straight pipes, but are easily observed in brass wind instruments. It can also be seen that the presence of the sand reduces the maximum amplitude by approximately an order of magnitude and shifts the location of strongest resonance away from the rim to a an area of the bell with much smaller bore diameter, illustrating the concept of spatially distributed resonance regions. The fact that the resonance frequencies are spatially distributed in the vibrating bell section in both the free and damped case, and that damping the wall vibrations significantly changes the resonance frequencies, is demonstrated by plotting the predicted resonance frequency versus the axial position for both cases as shown in Fig. 11.

IV. CONCLUSIONS

The similarity between the theoretical and experimental results indicate that a significant portion of the acoustical effects attributable to bell motion during play can be accounted for by assuming a axially symmetric motion of the wall that is caused by the internal air pressure. While these strain oscillations occur throughout the instrument, their effect is most pronounced when they occur in the bell section. This explains both the lack of observation of these effects in cylindrical pipes and the observation that damping parts of the instrument away from the bell produce no measurable effects.

The only significant difference between the theoretical predictions and the experimental measurements is that the frequencies at which the inversion of effects occur are predicted to be significantly higher than they actually are. This may be attributable to the fact that neither the rim wire of the trumpet nor the fold at the end of the bell were included in the model. At the end of the bell the metal is folded backward over a wire, which significantly changes the structural resonance in that area. Since the greatest effect attributable to wall vibrations occurs in that area, it is not surprising that the predicted resonance frequencies deviate from the measured values. However, given the qualitative similarity between the measurements and the model, it is likely that a more accurate model of the structure will result in better quantitative agreement. Including small variations from a perfectly cylindrical wall in the theory, which undoubtedly exist in real instruments, will also enhance the predicted effects. Adding the energy loss due to direct radiation from the moving wall may also be important. But even without such enhancements the theoretical predictions are surprisingly similar to the experimental results.

There is no longer any doubt that the vibrations of the bells of brass wind instruments affect the sound produced during play. The etiology of this effect is complicated and probably involves multiple phenomena; however, the work reported here indicates that the majority of the audible effects can be attributable to the presence of strain oscillations in the bell and their interaction with the air column of the instrument.

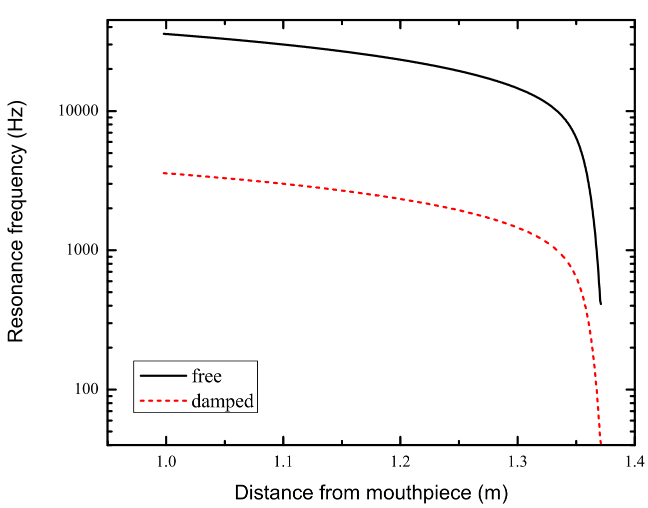

FIG. 11. (Color online) Theoretical resonance frequency of the strain oscillations in the bell section vs. the axial position.

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